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Eugen Rosca

Polytechnic University of Bucharest

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Mihaela Popa

Polytechnic University of Bucharest

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Florin Valentin Rusca

Polytechnic University of Bucharest

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TRANSPORT NETWORKS RELIABILITY AND VULNERABILITY IN LARGE URBAN AREAS

Eugen ROŞCA , Mihaela POPA, Florin RUSCĂ
eugen.rosca@upb.ro, mihaela.popa@upb.ro, florin_rusca@webmail.pub.ro

Eugen Roşca, Lecturer, Ph.D., Mihaela Popa, Associated professor, Ph.D., Florin Ruscă, Assistant
University Politehnica of Bucharest, Transportation Faculty
313 Splaiul Independentei, 060042 Bucharest
ROMANIA

Abstract: *The concepts of reliability and vulnerability are important when investigating the ability of transport networks to provide continuity in operation and maintaining the level of service between acceptable limits. The two concepts are discussed in a complementary way, outlining the specific features of each one. Reliability is described under connectivity, travel time and capacity aspects, whereas vulnerability is analyzed through the consequences of links or nodes failure, irrespective of the probability of failure, and mainly through changes of Hansen index of accessibility and users total cost. Using Monte Carlo computer simulation and TransCad software, the paper presents a methodology to assess connectivity reliability and vulnerability of congested road networks in large urban areas.*

Key words: *transport networks, reliability, vulnerability, accessibility, traffic congestion.*

RELIABILITY AND VULNERABILITY – INTERCONNECTED CONCEPTS

The concepts of reliability and vulnerability are quite important in assessing the ability of transport networks to provide continuity in operation. The natural disasters occurring during the recent years (earthquakes, floods, fires), the malevolence (terrorist acts, sabotages, wars), the spread out of the human habitat and mainly the extension of urban areas and traffic congestion on road networks provided a special interest in the researches on transport networks reliability and vulnerability. The impact of nodes or link disruption could be quite significant. The transport planners or policy makers need methods and decision support tools to evaluate threats to transport networks facilities and to assess the consequences of network functionality disruption and failure of its elements.

Economic, social and environmental benefits come from the possibility to evaluate, manage and minimize the impacts of transport networks degradation. At urban level, this is translated in reducing users transport costs, alleviation of traffic congestion and negative externalities, trade and social activities continuity.

The reliability of transport networks elements is a probabilistic measure that refers their ability not to fail or malfunction, during a specific period, given a set of performance guidelines. Even if some elements of the transport network are failed, the network could remain functional although with less performances.

One differentiates three forms of network reliability [1, 2]:

♦ *connectivity reliability* – the probability that two nodes in a network remain connected, i.e. there still is a path connecting them when a set of links have been cut off;

♦ *travel time reliability* – the probability that a trip between an origin and a destination node can be completed within a given time interval; the travel time can be affected by the imperfect knowledge of drivers and variation of link flows due to route choice decision;

- *capacity reliability* – the probability that a network can accomplish a given level of travel demand, i.e. the reserve capacity can accommodate the required demand for a specific capacity loss due to network degradation.

In contrast to reliability, the concept of vulnerability is related to the consequences of network elements failure, irrespective of the probability of failure. It is possible that a link failure may have a very small probability, but when the event occurs, the adverse social, economic and environmental impacts may have such an intensity to indicate a major problem. Vulnerability analysis provides a way to find structural weakness in the network topology that makes it vulnerable to consequences of failure or degradation. Taylor and D'Este [1] distinguish two forms of vulnerability in transport networks:

- *cost related vulnerability* – if the degradation of one or more links on a path connecting two nodes leads to substantial increase of the generalised cost of travel between them, then the connection between those nodes is vulnerable;
- *accessibility vulnerability* – a node is vulnerable if the failure of a small number of links in the network results in a severe decrease in the accessibility of that node.

CONNECTIVITY RELIABILITY

The probability that two nodes in a network remain connected can be computed by establishing the paths set between nodes and their reliability [2,3,4]. The reliability $P(\mathbf{X}_{ij})$ of a series of links $\mathbf{X}_{ij}=(ik, kl, ..., sj)$ connecting two nodes *i* and *j*, can be calculated as $P(\mathbf{X}_{ij}) = \prod_{kl \in \mathbf{X}_{ij}} p_{kl}$, where p_{kl} is the expected reliability of the link *(kl)*. If $p_{kl}=1$ the link is certainly operational and if $p_{kl}=0$ the link is certainly failed. The reliability of nodes is considered 1. Their disruption probability can be transferred to the adjacent links. Each network can be decomposed into minimal paths and minimal cuts set. A minimal path between two nodes is a series of links connecting the two nodes where a failure of a link results in the failure of the connection. For every minimal path A_{ij} connecting the nodes *i* and *j*, there is a logical function $\alpha(A_{ij})$ that takes the value 1 if the path is operational and 0 otherwise.

$$\text{(1)} \qquad \alpha(A_{ij}) = \bigcap_{kl \in A_{ij}} x_{kl} \, ,$$

where $x_{kl}=1$ if the link (*kl*) is functional and 0 if it is failed.

The minimal cut is a set of links where the recovering of a link results in recovering of the connection between the two nodes. For every minimal cut B_{ij} assigned to the pair of nodes *i* and *j*, there is a logical function $\beta(B_{ij})$ that takes the value 0 if all links included are failed and 1 when at least one link is operational.

$$\text{(2)} \qquad \beta(B_{ij}) = \bigcup_{kl \in B_{ij}} x_{kl} \, .$$

The network structure can be represented as a system of parallel minimal paths and serial minimal cuts. Because the links belong to several paths and cuts at the same time, the independence of the elements cannot be assumed in computing the reliability of the network. However, it is possible to calculate the lower and the upper bounds of the connectivity reliability for each pair of nodes (*i,j*):

$$\text{(3)} \quad \prod_{s=1}^{S} \left[1 - \prod_{kl \in B_{ij}^{s}} \left(1 - p_{kl}\right) \right] \le P(\mathbf{X}_{ij}) \le 1 - \prod_{q=1}^{Q} \left(1 - \prod_{kl \in A_{ij}^{q}} p_{kl} \right)$$

where S is the number of minimal cuts and Q the number of minimal paths for the pair of nodes (*i,j*).

For large networks, it is quite difficult to identify all minimal paths and cuts. Monte Carlo simulation is a more effective technique to evaluate connectivity reliability in large transport networks. The method uses the random generation of different states of the network and computes the probability of connection between each pair of nodes. For the network in Figure 1, where the probabilities of functioning are written above every link, the connectivity reliability is depicted in Table 1.

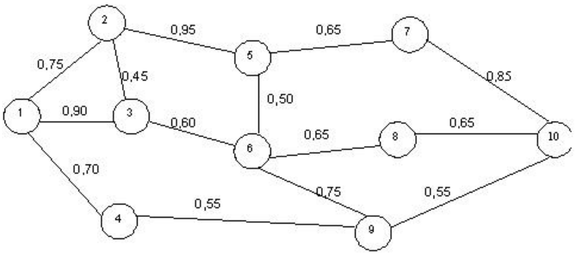
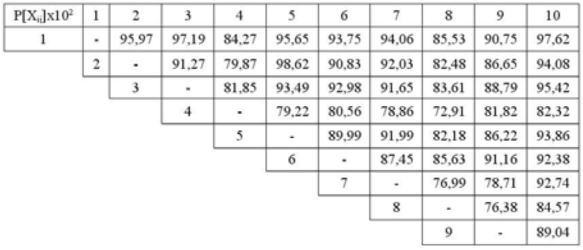


Fig. 1 Links functioning probabilities

Table 1 Network connectivity reliability



$P[X_{ij}]x10^2$	1	2	3	4	5	6	7	8	9	10
1	-	95,97	97,19	84,27	95,65	93,75	94,06	85,53	90,75	97,62
2		-	91,27	79,87	98,62	90,83	92,03	82,48	86,65	94,08
3			-	81,85	93,49	92,98	91,65	83,61	88,79	95,42
4				-	79,22	80,56	78,86	72,91	81,82	82,32
5					-	89,99	91,99	82,18	86,22	93,86
6						-	87,45	85,63	91,16	92,38
7							-	76,99	78,71	92,74
8								-	76,38	84,57
9									-	89,04

NETWORK VULNERABILITY

Accessibility related vulnerability

Taylor and D'Este [1] use accessibility and Hansen accessibility index to characterize transport networks vulnerability. The accessibility of a node *i* is

$$A_i = \sum_{j \neq i} B_j f(c_{ij}), \tag{4}$$

where B_j is the attraction measure of node *j*, c_{ij} represents the generalized cost of travel from node *i* to *j* and $f(c_{ij})$ the impedance function of the journey. Usually, the impedance function is the inverse of the generalized cost of travel (distance, time or money units) or a negative exponential function (e.g.: $f(c_{ij}) = e^{-\beta c_{ij}}$, where β is a calibration parameter).

The Hansen index of node accessibility is

$$HA_i = \frac{\sum_{j \neq i} B_j f(c_{ij})}{\sum_{j \neq i} B_j}, \tag{5}$$

and the accessibility index for the entire network is

$$TA = \sum_i HA_i . \tag{6}$$

An incident occurred in the network that causes the failure of the link *k* results in nodes and network accessibility decreasing:

$$\begin{aligned} \Delta HA_i &= HA_i^{(0)} - HA_i^{(k)}, \\ \Delta TA &= TA^{(0)} - TA^{(k)}. \end{aligned} \tag{7}$$

where the index (0) refers to the undamaged network and the index (k) to the network with the link *k* inoperable.

Relative variation of accessibility for nodes and the whole network could also be computed:

$$\begin{aligned} \%\Delta HA_i &= \frac{HA_i^{(0)} - HA_i^{(k)}}{HA_i^{(0)}}, \\ \%\Delta TA &= \frac{TA^{(0)} - TA^{(k)}}{TA^{(0)}}. \end{aligned} \tag{8}$$

Cost related vulnerability

Jenelius *et al.* [5] use, as a measure of reduced performance of the network, the increase in the generalized cost of travel (time, distance, money) for the users. When a link *k* is closed, the network may be divided into several disconnected parts, so that a number of trips from origin *i* are not able to reach the destination *j*. Thus results an unsatisfied demand

$$u_{ij}^{(k)} = \begin{cases} \varphi_{ij} & \text{if } c_{ij}^{(k)} = \infty \\ 0 & \text{if } c_{ij}^{(k)} \neq \infty \end{cases},$$

where φ_{ij} represents the travel demand from node *i* to node *j* and $c_{ij}^{(k)}$ is the generalized cost of travel from node *i* to *j* when link *k* is closed.

Therefore, there is a dichotomy of the link importance according to travel cost increasing and unsatisfied demand into the network. If the link *k* belongs to the set of non-cut links (L^{n-c}), the importance of the link *k* for the whole network is

$$\Omega(k) = \frac{\sum_i \sum_{j \neq i} \varphi_{ij} (c_{ij}^{(k)} - c_{ij}^{(0)})}{\sum_i \sum_{j \neq i} \varphi_{ij} c_{ij}^{(0)}}, \tag{9}$$

where $c_{ij}^{(0)}$ is the generalized cost of travel from node *i* to node *j* in the undamaged network.

The importance regarding the unsatisfied demand of a link *k* is

$$\Omega_{uns}(k) = \frac{\sum_i \sum_{j \neq i} u_{ij}^{(k)}}{\sum_i \sum_{j \neq i} \varphi_{ij}}. \tag{10}$$

In addition, the link disruption is translated into nodes exposure. The demand weighted exposure of node *i* is the maximum value over all non-cut links:

$$\Phi(i) = \max_{k \in L^{n-c}} \frac{\sum_{j \neq i} \varphi_{ij} (c_{ij}^{(k)} - c_{ij}^{(0)})}{\sum_{j \neq i} \varphi_{ij} c_{ij}^{(0)}}. \tag{11}$$

The exposure regarding the unsatisfied demand for the node *i* is

$$\Phi(i) = \max_k \frac{\sum_{j \neq i} u_{ij}^{(k)}}{\sum_{j \neq i} \varphi_{ij}}. \tag{12}$$

CASE STUDY – BUCHAREST INNER CITY ROAD NETWORK VULNERABILITY

Bucharest is a city with historical evolution as many of the European capitals. The road network includes a main route from North to South, having the greatest transit capacity, but also another one from East to West. The radial-circular graph depicted in Figure 2 is used for modelling the road network of such historical city. The graph links have various lengths, capacities and cost functions. Without affecting the generality, there are considered four length categories (l_0=0.5 km; l_1=1 km; l_2=1.5 km; l_3=2 km).

The streets are classified in four categories, starting from the current situation in Bucharest

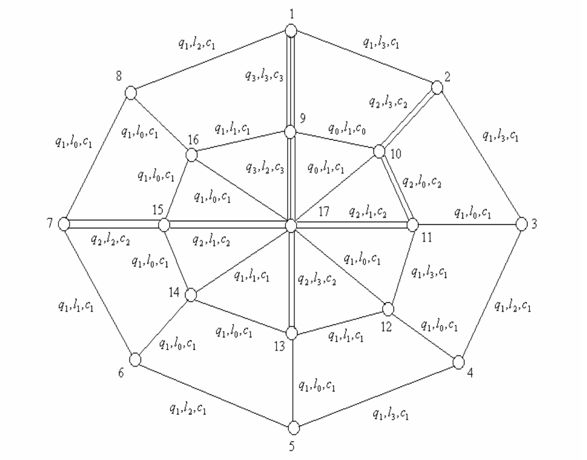


Fig. 2 Bucharest inner city radial-circular road network

(q_i – capacity, l_i – length, c_i – cost function)

The links characteristics are shown in Table 2.

Table 2 Network links characteristics

Type	Capacity [pcu/h/way]	Users' cost function [10^{-2} EUR/km/h]
Three or more lanes per way	6,000 (q_3)	$3+3.6(\varphi/q_3)^7$ (c_3)
Two lanes per way	4,000 (q_2)	$3.2+3.8(\varphi/q_2)^7$ (c_2)
One lane per way	2,000 (q_1)	$3.36+4.04(\varphi/q_1)^7$ (c_1)
One way	1,500 (q_0)	$3.36+4.04(\varphi/q_0)^7$ (c_0)

The users' costs function on link *i* is:

$$c_i = c_{0i} + \alpha(\varphi_i / q_i)^7 \ [10^{-2} \text{ EUR/km/h}], \quad (13)$$

where c_i is the utilization cost per km and hour for a passenger car unit (pcu) in average conditions; the exponent 7 was selected according to literature [6, 7], outlining external effects generated by car traffic; c_{0i} – the free flow utilization cost, related to the link type and capacity (influenced by the maximum legal speed) for a passenger car unit (pcu); α - a coefficient whose level balances the standard trip time value during the working day hours with the travel lost time value of an average driver in Bucharest [8], φ_i - traffic flow, q_i – link capacity.

The O-D matrix depicted in Table 3, provides the number of personal car unit during the average hour of the working day [pcu/h]. The trips are assigned on the network itineraries, using the successive average method. The solution provided is convergent to Wardrop's equilibrium [9].

Table 3 O-D trip matrix [pcu/h]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	-	200	100	-	100	-	300	200	500	-	300	-	-	100	100	-	1000
2	500	-	400	-	-	-	700	800	-	-	-	-	-	-	-	-	300
3	200	-	-	100	100	-	300	700	600	300	100	-	-	-	500	400	200
4	500	300	100	-	100	100	200	500	100	100	100	100	-	-	200	200	200
5	200	500	200	-	-	100	200	-	700	300	200	-	400	-	-	100	400
6	400	600	300	100	-	-	100	-	300	400	-	-	100	100	100	-	300
7	500	700	900	800	50	100	-	100	700	300	400	100	100	100	400	100	600
13	300	800	200	-	100	-	-	500	300	200	300	-	-	-	300	-	400
16	900	800	700	900	200	300	500	100	50	150	300	500	-	100	-	-	700

None of the links in the graph depicted in Figure 2 is a cut-off link, and the failure of a link does not generate unsatisfied demand. Therefore, links importance related to their weight in network vulnerability can be evaluated using eqn. 9.

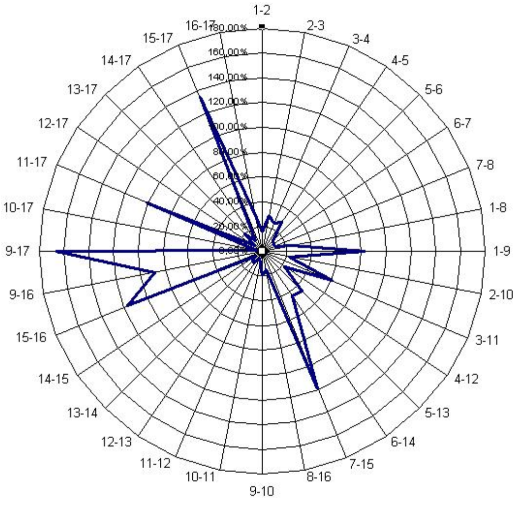


Fig. 3 Links importance regarding network vulnerability

Figure 3 depicts links importance regarding the network vulnerability $\Omega(k)$. The most vulnerable sections of the network comprise the links of the N-S and E-W corridors (9-17, 15-17, 7-15, 11-17). The failure of the link 9-17 increases users total costs by 166% and the failure of link 15-17 by 134%. Also, great importance presents the links 15-16 and 9-16 connecting the north and the west part of the city through a path that does not cross the city centre. These two sides of the city are the main beneficiaries of residential and commercial zones development during the last decade. They comprise important zones that generate and attract traffic. At present, the Bucharest Municipality provides funds for developing alternative routes that connect the north and the west part of the city. The current projects benefits of land opportunities and will contribute to traffic congestion reduction and minimization of road network vulnerability. Less importance present the links from the south and east part of the network. The south and the east part of the city are still emerging ones, but their advantages come from the land availability and prices. The future expansion in these zones, according to municipality and entrepreneurs (industrial, residential or commercial) strategies and to global markets guidelines, could provide new traffic flows generating and ending in these areas. Therefore, a new O-D matrix will be set up and changes in links importance are expected.

CONCLUSION

Transport network reliability and vulnerability demand an integrated approach. Both technical and non-technical factors are of great importance. Assessment methodologies based on multiple perspectives are recommended. Proactive measures are needed in order to prevent disruptions and to assure that the network will be able to maintain an acceptable level of service. It is important to prevent the network from failure, but if this occurs, it is also important to minimize the extent of the negative effects and to restore the normal state as quick as possible. The methods presented are useful for the urban planners, traffic engineers and policy makers in focusing their efforts in refining techniques for identifying network weakness, evaluating cost-effective risk management and remedial responses such as reducing risk profile, modernizing current infrastructure, creating alternative routes and minimizing socio-economic impacts in terms of location and accessibility to markets, services and facilities.

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НАДЕЖДНОСТ И УЯЗВИМОСТ НА ТРАНСПОРТНИТЕ МРЕЖИ В ГОЛЕМИТЕ ГРАДСКИ РАЙОНИ

Еужен РОШКА , Михаела ПОПА, Флорин РУСКА
eugen.rosca@upb.ro, mihaela.popa@upb.ro, florin_rusca@webmail.pub.ro

Еужен Рошка, преподавател доктор, Михаела Попа, доцент д-р, Флорин Руска, асистент
Университет „Политехника” в Букурещ, Транспортен факултет
313 Сплаилул Индепендентей, 060042 Букурещ
РУМЪНИЯ

Резюме: *Концепциите за надеждност и уязвимост са важни, когато се инвестира във възможността на транспортните мрежи да осигурят непрекъснатост на експлоатацията и поддържане на разнище на обслужване в приемливи граници. Двете концепции се обсъждат в чрез допълване, като се подчертават специфичните характеристики на всяка една. Надеждността се описва в аспекти като възможност за свързване, време на пътуване и капацитет, докато уязвимостта се анализира чрез последствията от прекъсването на връзките и възлите независимо от възможността за неуспех и главно чрез промените на индекса Хансен за общата стойност на достъпа и потребителите. Като се използва компютърното симулиране Monte Carlo и софтуера TransCad, докладът представя методология за оценка на надеждността за свързване и уязвимостта на задръстените пътни мрежи в големи градски райони.*

Ключови думи**:** *транспортни мрежи, надеждност, уязвимост, достъпност, задръстване.*