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Predictive Models for Routing in Urban Distribution

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Abstract

Freight urban distribution faces various and complex uncertainties in relation to travel and idle times in operation. In order to schedule the distribution, time reserves have to be factored in to compensate for the random travel time variations. This paper presents methodologies to estimate the time reserves based on travel and idle time costs, and statistical data. In addition there are solutions presented to stimulate on time deliveries to customers by giving priority to vehicles running on time according to the planned schedule. The comparison between alternatives for a certain origin and destination is performed with the aid of multi-criteria analysis. The problem of finding the optimal route is solved employing a specific methodology. Recommendations are made for on time deliveries.

Keywords: Vehicle routing; one-to-many distribution; traffic safety

1. Introduction

In the urban distribution, the optimum use of the delivery van is influenced by slow speeds in traffic, and significant idle times for most deliveries. We are referring here to the idle times caused by the lack of coordination in the reception of frequent deliveries from multiple producers and distribution centres.

Especially in the case of the delivery of controlled temperature goods, the idle times are counterproductive. Fuel consumption and emissions when the van is idle awaiting unloading are far greater than in operation, when

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performing the trip. In striking contrast with the requirements for a more sustainable transport, this is a frequent occurrence at the supermarkets in Romania during the summer months and busy times of the year. The contradicting interests of the supermarkets (avoiding the over dimensioning of unloading facilities) and the carrier (wishing to deliver anytime and unload right away) are not harmonized. In other words, there have not been established delivery standards for the facilities of the warehouse or supermarket, nor the delivery vehicles (Raicu, 2007).

The carriers seem to not realize that the waste congestion at delivery points generates because the efforts of some supermarkets to plan the arrival of deliveries had virtually no effect. Congestion persists and idle times in the queue waiting unloading are substantial.

Besides the carriers accepting this situation we also have to take into account objective causes. These relate to the difficulty of planning travel times, including estimating arrival time on the over congested urban road network for almost the entire duration of the day. Travel time affected by various uncertainties has to be treated as a random variable.

Two correlated objectives are proposed hereafter:

- Stimulate carrier to obey a plan of arrivals
- Carriers to more accurately estimate travel time

Both objectives have the same aim; to diminish delivery congestion, and obviously save resources (transport and energy).

2. Priority Service

In order to stimulate the carrier to obey a plan of arrivals, differential access to the unloading facilities is a gradually applied.

Initially, two priority classes are allowed. In the first class, includes all the vehicles that arrived prior to the planned arrival, and in the second class late arrivals are included. First class vehicles have priority for unloading over the second class.

Later, after the initial rule had effect by establishing a certain order it could extend the range of priority classes depending on the difference between planned and actual arrival time (highest priority to the minimum early arrival, and lowest priority for the latest arrival). Of course, a high number of priority classes would not work in practice.

In the case of a M/D/1 (Hall, 1990) model for example, for the relative priority of service (the demand from an inferior class *i* in service at the moment of arrival of a demand from a superior class *j* continues being served) the average waiting time for the category *j* demand is:

$$\bar{t}_{p,j} = \frac{\rho \cdot \bar{t}_{sv} \cdot \left(1+\upsilon_{sv}^2\right)}{2\cdot\left(1-\rho_{j-1}\right)\cdot\left(1-\rho_j\right)};\ \rho_{j-1} = \sum_{i=1}^{j-1}\rho_i\ ,\ \ \rho_j = \sum_{i=1}^{j}\rho_i\ ,\ \ \rho = \sum_{i=1}^{k}\rho_i\ , \tag{1}$$

where $\bar{t}_{sv}$ is average service time;

υ_{sv} - service time variation coefficient ;

$\rho_{j-1},\ \rho_j,\ \rho$ - different classes of demand, respective total demand ($\rho_j < 1, \forall j = \overline{1,k}$; $\rho < 1$);

k - number of priority classes.

In particular, for $k = 2$, results:

- for 1st priority class:

$$\bar{t}_{p,1} = \frac{(\rho_1 + \rho_2) \cdot \bar{t}_{sv} \cdot (1 + \upsilon_{sv}^2)}{2 \cdot (1 - \rho_1)} \tag{2}$$

- for 2nd class:

$$\bar{t}_{p,2} = \frac{(\rho_1 + \rho_2) \cdot \bar{t}_{sv} \cdot (1 + \upsilon_{sv}^2)}{2 \cdot (1 - \rho_1) \cdot (1 - \rho_1 - \rho_2)} \tag{3}$$

If the demands (vehicles arrived to unload) are served without differentiation (FIFO rule), under same conditions the average waiting time is:

$$\bar{t}_n = \frac{(\rho_1 + \rho_2) \cdot \bar{t}_{sv} \cdot (1 + \upsilon_{sv}^2)}{2 \cdot (1 - \rho_1 - \rho_2)}, \tag{4}$$

noting that $\bar{t}_{p,2} > \bar{t}_n > \bar{t}_{p,1}$, late vehicles will wait longer in the case of priority service. The late arrival incurs a penalty through longer waiting time.

The average waiting time for all demand in the case of priority service is:

$$\bar{t}_p = \frac{\lambda_1 \cdot \bar{t}_{p,1} + \lambda_2 \cdot \bar{t}_{p,2}}{\lambda_1 + \lambda_2} \tag{5}$$

λ_1 and λ_2 being the arrival intensities for the two categories of demand, which is the same as for the non-priority service case, $\bar{t}_n = \bar{t}_p$, because as it is known (Hall, 1990) the average waiting time is not dependent on the order (rule) of service.

The resource saving as to the current situation results from adjusting the attitude of the carriers by applying penalties for lateness. Thus, the uncertainties relating to the unloading of the facilities are reduced as well as avoiding system overload, when arrival rates are higher than the service rate.

3. Planning the Deliveries

3.1. Planning deliveries to account for travel time and waiting costs

As seen in Fig. 1, the travel time from an origin to a destination varies according to the density of the probability function $f(\tau)$ between a minimum value, τ_{min} and a maximum value, τ_{max}, the average value being τ_0. As to the planned arrival time t_s can differentiate between early and late arrivals.

If with a certain probability ω (see Fig. 1) the aim is to have an early arrival (before the planned time t_s), then the delivery time t_e has to be $\tau_0 + \Delta\tau$ before the arrival time, meaning a time reserve $\Delta\tau$ has to be added to the average travel time τ_0 .

If all late arrivals are excluded ($\omega = 0$), then the delivery time would be set according to the maximum travel time, τ_{max}, and the time reserve would be $\Delta\tau = \tau_{max} - \tau_0$.

However, because such a strategy results in the poor use of resources, an adequate measure of the time reserve $\Delta\tau$ as a function of the early or late arrival costs has to be found. The minimum of the mathematical expectancy of the sum of travel and idle time costs is the criterion used to find the recommended time reserve $\Delta\tau$.

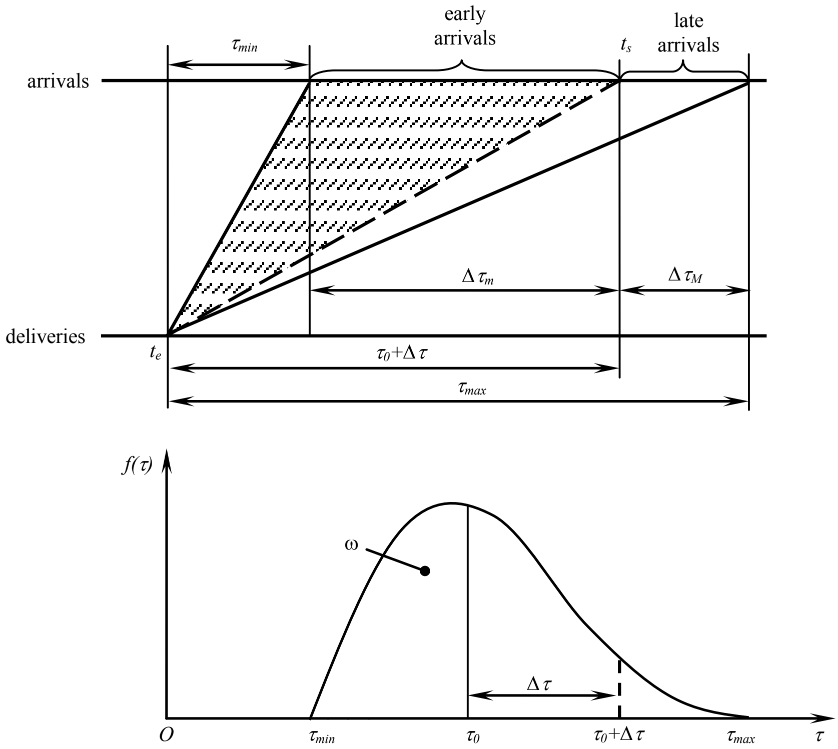

Fig. 1. Delivery time t_e, as a function of the estimated travel time $\tau_0+\Delta\tau$.

In the case of early arrival the mathematical expectancy of the sum of travel and idle time costs is:

$$C_a = \left[c_a \cdot \frac{\tau_{\min} + \tau_0 + \Delta\tau}{2} + c_s \cdot \left(\Delta\tau_m + \overline{t_{p,1}} \right) \right] \cdot \omega \ ; \ \ \Delta\tau_m = \tau_0 + \Delta\tau - \tau_{\min} \tag{6}$$

where c_a, c_s is the travel/idle cost per time unit;
$\overline{t_{p,1}}$ - the average waiting time in the case of priority service for 1st class (equation 2).

From (6) results the early arrivals have been distinctly included. It is only after the time t_s the demand is included in the 1st class of priority service, with the average waiting time $\overline{t_{p,1}}$. Thus, the adherence to the planned arrival time is stimulated because the early arrivals equate to certain idle times.

Similarly, for late arrivals the mathematical expectancy of the sum of travel and idle time costs is:

$$C_t = \left[c_t \cdot \frac{\tau_0 + \Delta\tau + \tau_{\max}}{2} + c_s \cdot \overline{t_{p,2}} \right] \cdot (1 - \omega) \tag{7}$$

where c_t is travel cost, per time unit for late arrival;
$\overline{t_{p,2}}$ - the average waiting time in the case of priority service for 2nd class (equation 3).

The total cost, for early and late arrivals, is:

$$C = C_a + C_t, \quad (8)$$

$$C = \left[c_a \cdot \frac{\tau_{\min} + \tau_0}{2} + c_s \cdot (\tau_0 - \tau_{\min}) + c_s \cdot \overline{t_{p,1}} - c_t \cdot \frac{\tau_0 + \tau_{\max}}{2} - c_s \cdot \overline{t_{p,2}} \right] \cdot \omega + \left(\frac{1}{2} c_a + c_s - \frac{1}{2} c_t \right) \cdot \Delta\tau \cdot \omega + \frac{1}{2} c_t \cdot \Delta\tau + \left(c_t \cdot \frac{\tau_0 + \tau_{\max}}{2} - c_s \cdot \overline{t_{p,2}} \right) \quad (9)$$

and using the *a, b, c, d,* notation in this particular order to simplify the equation, results:

$$C = a \cdot \omega + b \cdot \omega \cdot \Delta\tau + c \cdot \Delta\tau + d \quad (10)$$

noting that $\omega = f(\Delta\tau)$.

From $dC/d(\Delta\tau) = 0$ results:

$$a \cdot \omega' + b \cdot \omega' \cdot \Delta\tau + b \cdot \omega + c = 0 \quad (11)$$

After integrating the differential equation above, determining the integration constant for $\omega = 1$, when $\Delta\tau = \tau_{max} - \tau_0$, results in the optimum value ω_0 needed to ensure vehicles arrive before the planned time t_s:

$$\omega_0 = \frac{(b + c) \cdot [a + b \cdot (\tau_{\max} - \tau_0)]}{b \cdot (a + b \cdot \tau_0)} - \frac{c}{b} \quad (12)$$

Can easily observe that for $\Delta\tau_0 = \tau_{max} - \tau_0$, results $\omega_0 = 1$, meaning late arrivals are excluded.

From (12) also results the optimum value for the recommended time reserve $\Delta\tau_0$, which needs to be added to the average travel time τ_0 in order to achieve with ω_0 probability arrivals before the planned time t_s :

$$\Delta\tau_0 = \frac{a \cdot (1 - \omega_0) + (c + b) \cdot (\tau_{\max} - \tau_0)}{c + b \cdot \tau_0} \quad (13)$$

Coming back to the initial notations and assuming same travel costs, so $c_a = c_t = \gamma$ results:

$$\Delta\tau_0 = \left\{ \left[c_s \cdot \left(\tau_0 - \tau_{\min} + \overline{t_{p,1}} - \overline{t_{p,1}} \right) - \frac{1}{2} \gamma \cdot (\tau_{\max} - \tau_{\min}) \right] \cdot (1 - \omega_0) + \left(\frac{1}{2} \gamma + c_s \right) \cdot (\tau_{\max} - \tau_0) \right\} \cdot \frac{1}{\frac{1}{2} \gamma + c_s \cdot \omega_0} \quad (14)$$

For $\omega_0 = 1$, results $\Delta\tau_0 = \tau_{max} - \tau_0$.

Examining (14) results the $\Delta\tau_0$ value depends on:

- travel time values (τ_{min}, τ_0, τ_{max});
- travel and idle costs (c_a, c_t, c_s);
- the unloading facility operation ($\rho_1, \rho_2, \overline{t_{p,1}}, \upsilon_{sv}$) which determines the average waiting times ($\overline{t_{p,1}}$, $\overline{t_{p,2}}$);
- the probability of not arriving late, ω_0 (this does not figure in the equation for $\Delta\tau_0$ when the density probability function of the average travel time $f(\tau)$ is known).

Obviously, this type of estimation for the travel time $\tau_0 + \Delta\tau_0$, to avoid late arrivals, after the planned t_s with a ω_0 probability, based on travel and idle costs and the operation of the unloading facilities at the destination assumes the involved variables are known and implies a certain volume of calculations.

Calculations, even in the uniform distribution of the travel times case, with $\omega = f(\Delta\tau)$ of the type:

$$\omega = \frac{\tau_0 + \Delta\tau}{\tau_{\max} - \tau_{\min}}, \tag{15}$$

examining (14), results are not easy.

User friendly interface software would facilitate the calculations for different scenarios. However, remains the task of the user to correctly estimate the variables involved.

3.2. Planning deliveries based on statistical data

When the user does not have the data required to apply the previous model, but has statistical data regarding delivery and travel times an easier methodology can be applied to avoid substantial waiting times at the destination.

If t_e and t_s are the delivery times, respectively arrival to destination ($t_s - t_e = \tau_0$), then Δt from the planned delivery time t_e and travel time $\Delta\theta$, determine the ΔT from the planned arrival time, t_s (Fig. 2).

Suppose the empirical functions of the density probabilities for Δt and $\Delta\theta$ are known, respectively:

$$\Delta t \begin{pmatrix} \Delta t_i \\ f(\Delta t_i) \end{pmatrix} \text{ and } \Delta\theta \begin{pmatrix} \Delta\theta_j \\ \varphi(\Delta\theta_j) \end{pmatrix} \tag{16}$$

where $f(\Delta t_i)$ and $\varphi(\Delta\theta_j)$ are the relative frequencies of occurence of the Δt_i, respectively $\Delta\theta_j$ values.

Aswe are referring to a certain delivery for which we assume the variations Δt_i of the planned delivery time are not that significant to alter the conditions of travel, we can assume the two variables Δt and $\Delta\theta$ are independent.

Under these conditions, substituting the relative frequencies $f(\Delta t_i)$ and $\varphi(\Delta\theta_j)$ with probabilities $P(\Delta t_i)$ and $P(\Delta\theta_j)$ of occurence of the values Δt_i, respectively $\Delta\theta_i$, the probability of occurence of a variation ΔT from the planned arrival time, t_s, is:

$$P(\Delta T) = \sum_{\Delta t_i + \Delta\theta_j = \Delta T} P(\Delta t_i) \cdot P(\Delta\theta_j) \tag{17}$$

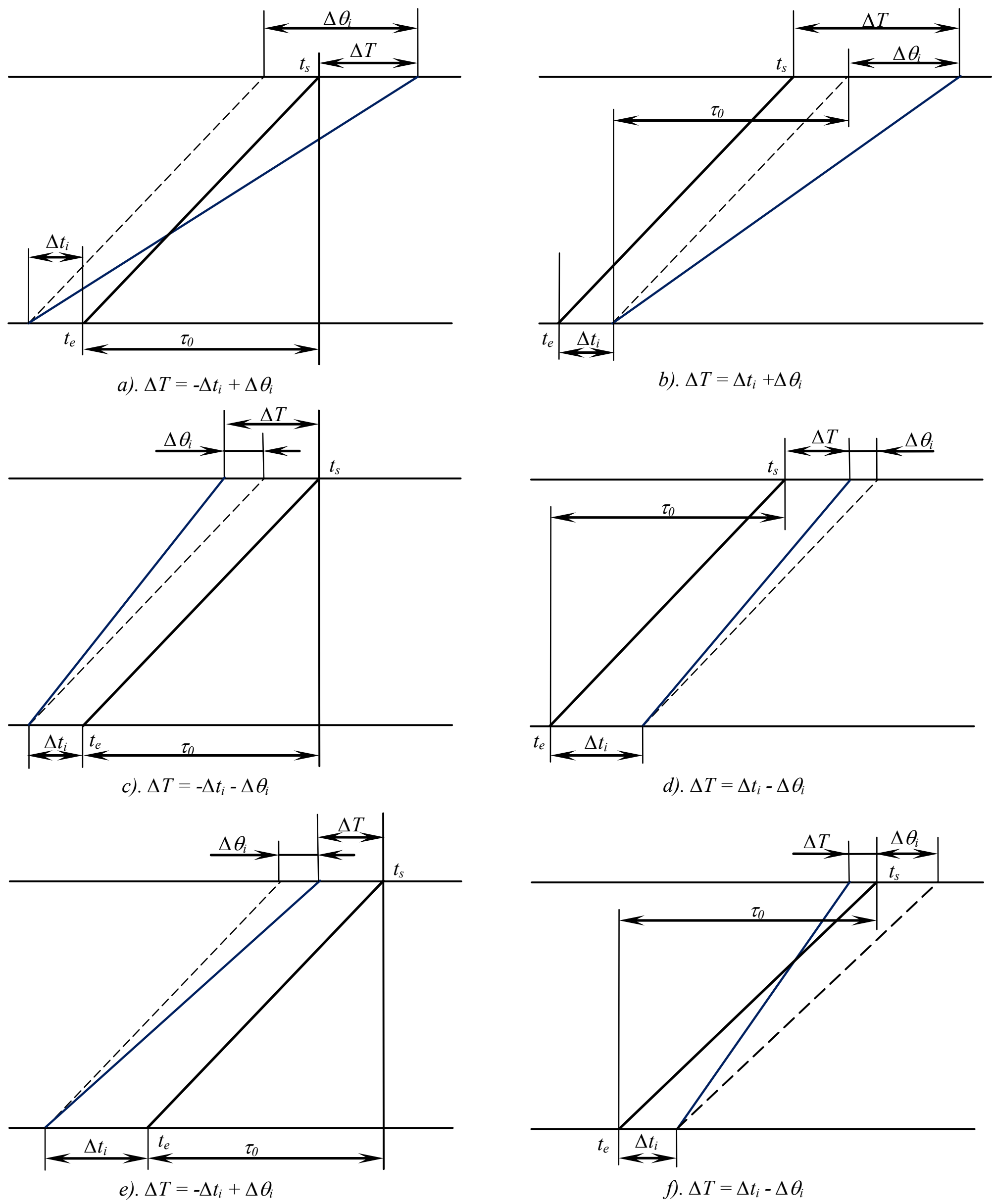

Fig. 2. Variation ΔT from arrival time as a function of the variations Δt_i, $\Delta \theta_i$ of the delivery time, respectively travel time

Calculations for all the $P(\Delta T)$ values leads to the cumulative density probability function $\Omega(\Delta T)$. In Fig. 3, the $\Omega(\Delta T)$ function is continuous. It is obvious though that given the way is obtained (from empirical functions $f(\Delta t_i)$ and $\varphi(\Delta \theta_j)$, discrete) the $\Omega(\Delta T)$ function can only be discrete.

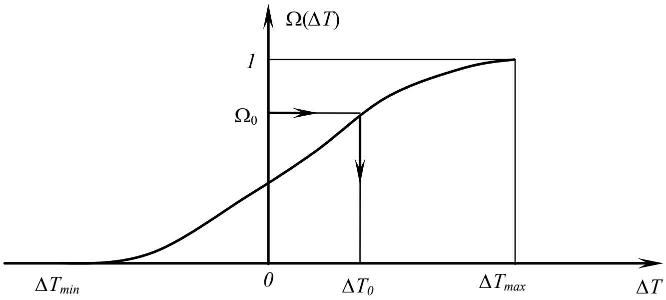

Fig. 3. Density probability function of the variations ΔT from the planned arrival time at destination t_s

From (17) and Fig. 3, results we can see that late arrivals with probability $\Omega(0) = P(\Delta T \leq 0)$ are excluded when the maximum variation, $\Delta\theta_{max}$, from the travel time is compensated by early delivery, before t_e with $|\Delta t| = \tau_{\max} - \tau_0$, the value of the maximum time reserve mentioned before.

Similarly we can determine the maximum variation ΔT_0 for late arrival:

$$|\Delta t_0| = \Delta T_0 - \theta_{\max} \tag{18}$$

This means that if the delivery is made before the planned time, t_e, with $|\Delta t_0|$, then with $\Omega_0 = P(\Delta T \leq \Delta T_0)$ probability the arrivals will be early, before the planned time t_s or with a maximum positive variation ΔT_0 .

4. Choosing the Route and Travel Time Estimation

In urban distribution, in highly connected networks there are multiple routes between an origin and a destination. Route choice is performed based on route length, travel time and cost (Bellman, 1958; Taniguchi, Thompson, Yamada & van Duin, 2001). Taking into account the dependency between route length and travel time, route length is mostly used. It is to be noted that the length used to rank the choices is not the length obtained from summing up the length of the arcs that make up the route. Choosing between the possible route options has to be done in relation to an equivalent length that is taking into account the route characteristics which substantially influence the travel time. We are taking into consideration the road type, number and characteristics of intersections, number of left turns, average road occupancy, temporary or permanent speed restrictions, etc. (Dijkstra, Drolenga & van Maarseveen, 2007).

Obtaining an equivalent length that takes into account all these characteristics is difficult and marked by uncertainties. This should look at extending the travel time due to the above mentioned characteristics. In other words, use the travel speed in the absence of all those elements which impede on traffic continuity and stability.

An easier ranking of the possible routes can be achieved with multi-criteria analysis based on the particularities of each route. Thus, if for each attribute $\underline{c}$ specific to the option $\underline{s}$ for the transport between an origin O and a destination D, the C_{cs} value is allocated, then by reuniting the criteria can calculate an aggregated index (a score) for each option s (Dijkstra, 2011):

$$I_s = \sum_{c=1}^{C} \alpha_c \cdot I_{cs} \tag{19}$$

where C is number of criteria included in the analysis;
α_c - weighting coefficient for the criterion $\underline{c}$;
I_{cs} - normalized score (index) as a function of $\underline{c}$ criterion for option $\underline{s}$.

The normalized index I_{cs} is reporting the value of C_{cs} on a 0,..., 1, scale:

$$I_{cs} = \frac{C_{cs} - \min_s\{C_{cs}\}}{\max_s\{C_{cs}\} - \min_s\{C_{cs}\}} \tag{20}$$

where $\min_s\{C_{cs}\}$, $\max_s\{C_{cs}\}$ are the minimum values, respectively maximum of the $\{C_{cs}\}$ values allocated to all the $s = \overline{1..S}$ attributes for the $\underline{c}$ criterion.

The ranking after I_s of all the possible route options S, corresponds to the preference order of the decision maker for performing the delivery in a certain way. If when choosing the route in a "one to another" distribution as a function of I_s the calculation of an equivalent route length is avoided, in a "one to many" distribution as well as when travel time has to be estimated, finding an equivalent route length is absolutely necessary.

Knowing the equivalent length of a route, travel time is calculated as a function of the maximum travel speed $V_{\max} = \min(\overline{V},\ V_t)$, where $\overline{V}$ is the maximum legal speed of the route, and V_t is the maximum speed a vehicle can reach with a full load.

In this fashion, when estimating travel time, the average travel speed specific to each route does not figure. In the mean time we have to mention that as a function of the same $V_{\max}$, the delays produced by the route characteristics mentioned before are assimilated with supplementary route lengths. For example, if in an intersection, the average number of vehicles retained in the intersection, in the direction corresponding to the analyzed route is β and the average time in intersection is Δt, then, due to that intersection the route length is adjusted so $\Delta l = \beta \cdot \Delta t \cdot V_{\max}$. The Δl calculation is based on average values of β, corresponding to the average traffic flow values on the arteries converging into the intersection and Δt, corresponding to the daily variation of the traffic lights cycle and phases, and also the braking and starting characteristics of the vehicle. As the values of β and Δt are for a certain time of the day when the transport scheduling is done these are characterized by relatively small dispersions, using the average value is not introducing significant errors in establishing the equivalent route length, travel time, respectively.

Hereafter, all references to the location of the urban distribution centre and customers suppose equivalent distances.

5. Choosing the Route in "one to many" Distribution

The routing problem analyzed herein looks at all customers B_i needing to be visited following the arc of the network on the right hand side, the direction of the traffic. The return maneuver on any arc of the network is not possible (the two traffic directions are physically separated) (Janssen, 1991).

The location of the B_i customers and T depot on the urban road network is shown in Fig. 4. Only the roads free from freight restrictions have been included. The nodes of the network are simple intersections except for node 7 which is a roundabout. All the 6 customers B_1, B_2,…, B_6, represented in Fig. 4 have to be visited in same trip. The distribution vehicle is hired and after visiting the last customer and does not have to return to the depot T. It can be directed depending on the location and opportunities for other jobs.

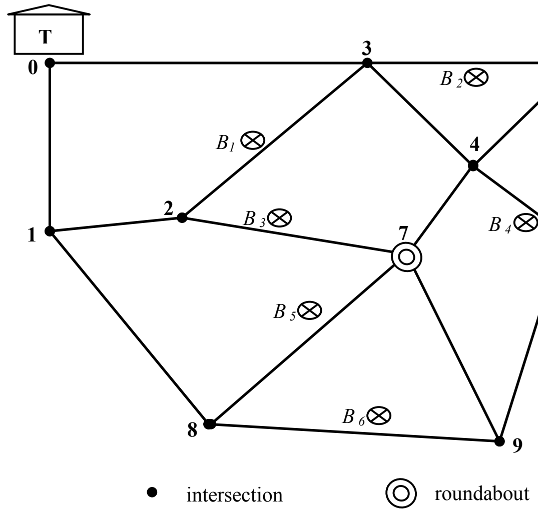

Fig. 4. Network with customer and depot location

Although the graph associated with the urban network is not an oriented graph, the requirement to visit each customer B_i (on the right hand side of the road) leads to the allocation of customers to oriented arcs, respectively $B_1 \in (3 \to 2)$; $B_2 \in (3 \to 5)$; $B_3 \in (7 \to 2)$; $B_4 \in (4 \to 6)$; $B_5 \in (7 \to 8)$; $B_6 \in (9 \to 8)$.

This means that the matrix ([S_{ij}] of the equivalent minimum length between all the nodes of the network (including 0, which is considered the node allocated to T), the B_i centres can be replaced with the nodes of the network which ensure access to the correct direction. For example, to visit B_1 and B_2, via node 3, for B_3 via node 7 etc., and the departure from B_1, via node 2, the departure from B_2 via 5, respectively from B_3 via node 2 etc.

The roundabout in node 7 means that the possible access to B_3 and B_5 and via nodes 2, respectively 8 has to be taken into account when looking at equivalent minimum length arcs.

The matrix ([S_{ij}] of equivalent minimum length between the nodes of the network can be replaced by the matrix ([S'_{ij}] of minimum routes between T and B_i (Table 1). The location of each B_i in relation to the access node is required to obtain ([S'_{ij}] from ([S_{ij}].

Table 1. Routes of minimum equivalent length ([S'_{ij}].matrix).

To / *From*	T(0)	B_1(3)	B_2(3)	B_3(7/2)	B_4(4)	B_5(7/8)	B_6(9)
T(0)	-	10	8	$13_{(2\text{-}7)}$	11	$13_{(7)}$	18
B_1(2)	8	-	13	9	10	9	14
B_2(5)	14	12	-	10	7	10	14
B_3(2)	7	17	9	-	12	$15_{(7)}$	20
B_4(6)	16	14	17	13	-	$13_{(7)}$	8
B_5(8)	11	21	17	16	17	-	21
B_6(8)	12	17	18	17	14	11	-

The equivalent distances from each entry node to B_i, respectively from B_i to the exit node, are used in the calculation of the ([S'_{ij}] matrix. For the case when the vehicle performs the distribution for all the centres, B_i, $(i = \overline{1..6})$ (starting from T and returning to T, after visiting the last centre) the minimum equivalent route has been calculated (Fig. 5). An algorithm specific to dynamic programming has been used. Based on Bellman's principle (Bellman, 1957, Cormen et al. 1990), the algorithm performs the process in the opposite sense of its evolution and identifies amongst successive sequences potential optimal strategies (marked with thick grey lines in Fig. 5). Arriving at the origin T and following the process in its evolution sense are the optimal strategy results (marked with a double grey-black line). These can be obtained by moving from one centre to the next, and from one sequence to the next, following the thick grey lines, in other words the potential optimal strategies.

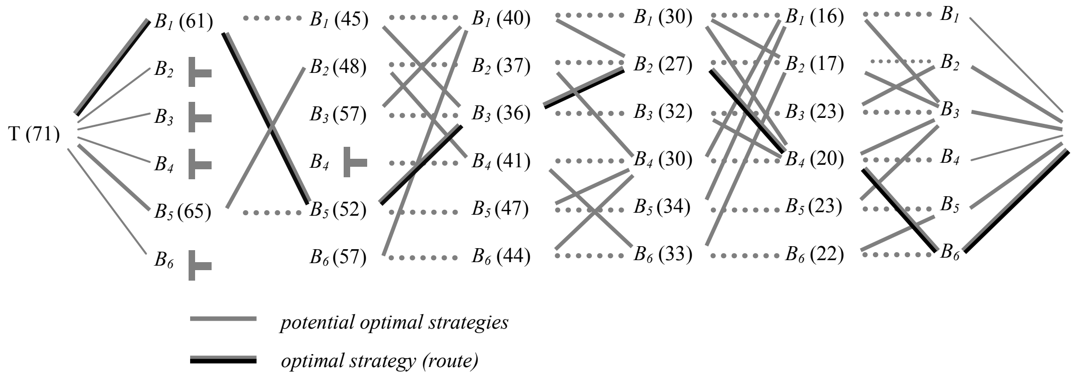

Fig. 5. Minimum equivalent length route

The optimal route (Fig. 5) has the following trajectory: T – B_1 – B_5 – B_3 – B_2 – B_4 – B_6 – T, with the equivalent length 71. Also from Fig. 5 we can identify the trajectory: T – B_5 – B_2 – B_4 – B_6 – B_1 – B_3 – T, with an equivalent length 78, which is greater than the previous one.

The two routes, represented in Fig. 6 show that certain arcs of the road network are used two or three times (case of arc (2-7), in the sub-optimal case). This means that those arcs used more than once can be the source of additional uncertainties in estimating the travel time if the adjacent arc or nodes have a high road safety risk. When ranking the route choices, a criterion like this is taken into account especially when the difference in total equivalent length is not significant.

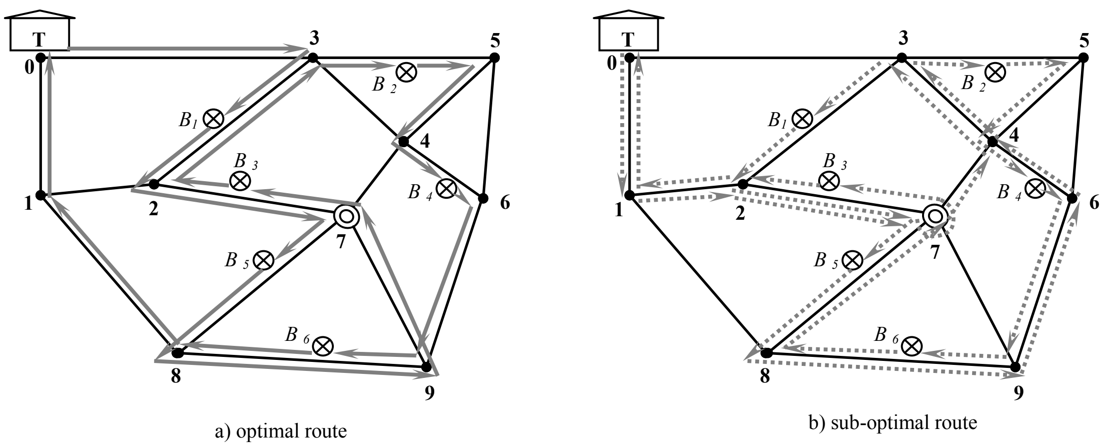

Fig. 6. The two routes of minimum equivalent length

If the (2, 7) arc, for example has a high road safety risk, then the B_3 customer can be left last to be served. An option like this means a greater equivalent route length can be recommended if the time windows required to reach the customers have to be strictly obeyed.

Suppose the route includes *m* customers to be visited in one trip. The probability of occurrence of a variation ΔT_1 from the planned arrival time at customer 1 is:

$$P(\Delta T_1) = \sum_{\Delta t_{i,T} + \Delta\theta_{j,(T-1)} = \Delta T_1} P(\Delta t_{i,T}) \cdot P(\Delta\theta_{j,(T-1)}) \tag{21}$$

where $P(\Delta t_{i,T})$ is probability of occurrence of the variation $\Delta t_{i,T}$ from the planned delivery time from depot T;

$P(\Delta\theta_{j,(T-1)})$ - probability of occurrence of variation $\Delta\theta_{j,(T-1)}$ from the travel time from T to customer 1.

The probability of occurrence of a variation ΔT_2 from the planned arrival time at customer 2 is:

$$P(\Delta T_2) = \sum_{\Delta T_1 + \Delta\upsilon_1 + \theta_{j,(1-2)} = \Delta T_2} P(\Delta T_1 + \Delta\upsilon_1) \cdot P(\Delta\theta_{j,(1-2)}) \tag{22}$$

where $\Delta\upsilon_1$ is the variation from the planned waiting time at customer 1

Generalizing, for customer *k*, the probability of the variation ΔT_k from the planned arrival time is:

$$P(\Delta T_k) = \sum_{\Delta T_{k-1} + \Delta\upsilon_{k-1} + \theta_{j,((k-1)-k)} = \Delta T_k} P(\Delta T_{k-1} + \Delta\upsilon_{k-1}) \cdot P(\Delta\theta_{j,((k-1)-k)}) \tag{23}$$

where ΔT_k can be written as:

$$\Delta T_k = \Delta t_{i,T} + \sum_{s=1}^{k-1} \Delta\upsilon_s + \sum_{s=1}^{k-1} \Delta\theta_{j,((s-1)-s)} \tag{24}$$

If we take into account the waiting times at customers corresponding to a normal distribution, then for a fair number of customers $\sum \Delta\upsilon \cong 0$ and essential to compensate the variations from the travel times, is finding the value of $\Delta t_{i,T}$, ($\Delta t_{i,T} < 0$).

Conclusion

The carriers performing urban distribution can feel the effects of poor vehicle utilisation. Additional idle time at customers, waiting for processing and low speeds in congested and high risk road safety traffic networks, are the causes of reduced efficiency in the use of human and material resources.

Reducing the additional idle times caused by the lack of coordination of arrivals to a certain customer can be achieved through actions meant to stimulate the adherence to schedules. In this respect the differentiated treatment of arrivals has been proposed. Vehicles arriving on time are served with priority. The others will incur additional waiting time. In time this measure will lead to a better adherence to the planned schedule of arrivals to customers.

In order to meet a certain arrival to a destination constraint, in the conditions of travel time uncertainty, and in the planning process, a time reserve for the average travel time duration is proposed. The value of this reserve is calculated based on the travel and waiting costs for a density of probability function of the travel time. If the carrier does not have all the data to perform the calculation, he can set the delivery time based on statistical data.

In urban distribution the high degree of connectivity of the network presents the carrier with multiple route choices. For this a multi-criteria analysis of the route characteristics has been proposed.

The conclusions from the “one to another” distribution are excluded in the “one to many” distribution. Finding the optimal route to visit the customers in a single trip, as to an equivalent route length has been performed following a specific methodology.

Recommendations are being made for the additional analysis of the routes resulting after employing the minimum equivalent length route criterion, and for estimating the delivery time from the distribution centre. The consequences of routing including returning to the distribution centre are compared to routing including directing the vehicle to another destination after it has visited the last customer.

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