

## COLLABORATIVE SYSTEMS IN URBAN LOGISTICS

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*The paper presents a practical approach for the determination of collaborative solutions within an urban distribution logistics. Solutions to optimize the supply activities of distribution centers from suppliers are presented. Using a heuristic algorithm the minimization of the total distance covered by the trucks is realized by selecting the most appropriate combination of direct and through consolidation centers, hubs, supply, in strict compliance with the conditions imposed for supply/delivery. Consolidation realized within the consolidation centers (hubs) placed between suppliers and distribution centers is accompanied by the sharing of resources (warehouses, equipment, means of transport, staff) of the various actors involved in the supply/distribution. From here, the so called collaborative or mutual name given to this type of logistics. Optimization achieved by sharing the resources, is being treated for a specific situation - the supply of commercial companies' large stores in Bucharest.*

**Keywords:** supply collaborative systems; hybrid structure; hub consolidation; heuristic algorithm

### 1. Introduction

Managing the flow of freight between suppliers of consumer goods and distribution centers, and between them and stores requires a significant effort to optimize logistics as such problems involve a large number of variables, and so, using heuristic methods for organizing and operation of these systems occurs as an indicated approach [7, 8, 10].

Designing and organizing of a regional supply system with road vehicles significantly influence its performances. There are two classes of well-known theoretical models of supply: direct supply and distribution with hub consolidation [2,8].

In an exclusively direct supply system, each actor operates independently, with its own resources, in order to distribute goods to customers. Each shipment is delivered to a single customer in a single transport relation (Fig.1). Its use is

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justified especially in the following cases: (i) -distributions "just-in-time" or with deadlines; (ii) - when the logistic characteristics of the goods transported require freight to circulate isolated (for example, dangerous goods or that could contaminate other goods); (iii) the size of the expedition is sufficient for the transport vehicles (trucks) to be used at their maximum capacity.

In all other cases, the collaborative methods are used so that transportation costs would be reduced by consolidating goods belonging to more customers in the same vehicle, whose route reaches several locations, in some conditions (the total volume of freight to be supplied does not exceed the vehicle capacity, not to impose fixed supply times, and the goods that are loaded together have no risk of association).

The most common types of collaborative distributions are:

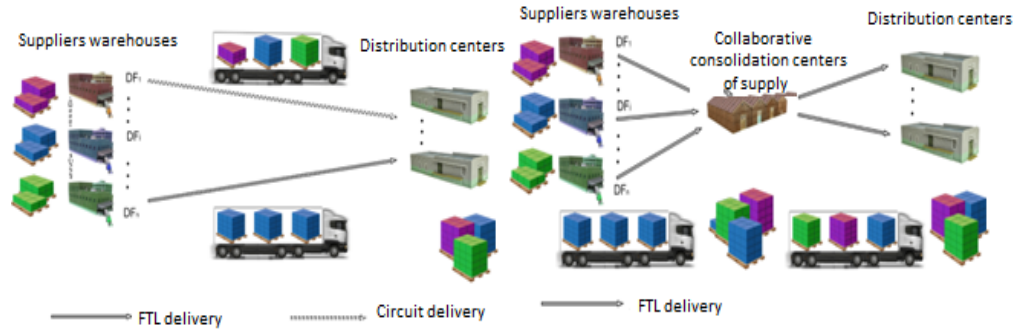
- *Direct delivery with pre-established/fixe schedule, also called "with vehicles in circuit"* which chooses the optimal route in terms of cost, problem which is solved, theoretically and practically, by vehicle routing (order of visitation) and of the shortest, or earlier, or, generally, the lowest cost path in the transport network (Fig. 1). Delivering with loading and distribution program over some fixed locations involves planning of fixed time intervals allocated to replenishment/delivery, but in terms of the variability of goods quantities. Such a problem can take different forms, depending on customer requirements, network restrictions, vehicle capacity, time interval for supply, number of stops and their related time, etc. [3, 5, 7, 8].

- *Consolidation center/hub distribution*: this is the case where there are several suppliers/producers located in the same geographic area, and especially when they also have common customers; the goods from suppliers/producers are concentrated and consolidated within a consolidation center, and then distributed to clients or their distribution platforms in complete road vehicles, or through modes of mass transport (rail and inland waterways), when a hub-and-spoke structure could be achieved. (Fig.2) [2, 10].

However, in case the customers of a particular supplier are in close geographical proximity to it, and the amount of goods is large enough to ensure full load of the vehicle, direct supply or eventually with vehicles in circuit is right solution, otherwise, the supply with consolidation center/hub is more appropriate.

In reality, there is a large orders variation due to: (i) -customers' demand frequency for products can differ and not observing any "pattern"; (ii) - the amount requested by them can also be very different from one order to another, so, arriving to wide variations of the transport service; (iii)- distances between suppliers and customers change together with the quantities of goods to be delivered, so the opportunity of using one of the two systems is customized depending on the situation. Under these circumstances, the need for designing a

hybrid supply system occurs which has a structure with consolidation hub, completed with direct distributions made simultaneously when justified.

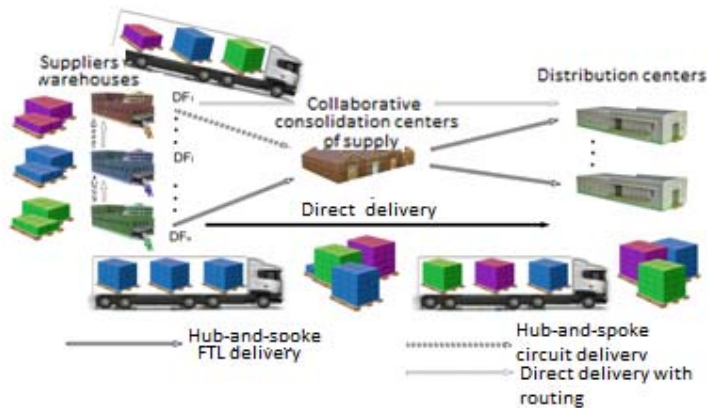


(Source: after [10])

Fig. 1. Direct delivery, or exclusive direct, or with vehicles in circuit

Fig. 2. Exclusive collaborative distribution with one consolidation hub

The paper determines a hybrid supply structure for the transport of general goods from manufacturers (suppliers of consumer goods) located in the metropolitan area of Bucharest to customers (warehouses/supply platform of large stores) in a heuristics approach, in order to use advantages offered both by a direct supply, exclusive and/or in circuit, and also by the consolidation hub distribution (Figure 3).



(Source: after [10])

Fig. 3. Simplified representation of a hybrid supply system

In the next section the research method is presented along with the description of the optimization problem and with the modeling hypothesis and heuristics solving algorithm. Section three presents a case study for a sequence of

supply demands in the Northern Western metropolitan area of Bucharest, from 20 centers of production/delivery to four storage/sales centers, for which, using the described algorithm, a hybrid solution is determined. In the final section discusses the results and the most relevant conclusions are drawn.

## 2. Research method

### 2.1. Supply problem formalization and modeling hypothesis

Modeling the supply system starts with the demand network represented by a graph (network) undirected  $G = (V, A)$ , with components described as follows:

-The set of nodes in graph,  $V = \{v_0\} \cup Vf \cup Vc$ , contains,  $v_0$ , the given location of collaborative consolidation center together with  $Vf = \{v_1, v_2, \dots, v_m\}$ , the set of known locations of the  $m$  suppliers of goods and the  $n$  known locations  $Vc = \{v_{m+1}, v_{m+2}, \dots, v_{m+n}\}$  of clients (metropolitan supply centers of a retail industry logistic chain). We associate to the  $G$  graph a matrix of the minimum distances between the nodes of the graph  $d_{ij}$ ,  $v_i$  and  $v_j$  for  $i, j = 0, 1, \dots, m+n$ . This minimum distances matrix is symmetric and satisfies the triangle inequality [6], so:

$$d_{ij} = d_{ji}, \text{ and } d_{ij} + d_{jk} \geq d_{ik}, \text{ for every } i, j, k = 0, 1, \dots, m+n.$$

The minimum distances,  $d_{ij}$ , are obtained by the minimum path within the uncongested transport network [9].

-The set of relations  $A = \{(v_i, v_j) \mid i = 1, 2, \dots, m; j = m+1, m+2, \dots, m+n\}$ , represents the set of transport demands which are revealed in a certain time; it consists of oriented arcs.

We associate to every arc  $(v_i, v_j) \in A$  a non-negative parameter representing demand,  $q_{ij}$  as the goods quantity necessary to be delivered from supplier  $v_i$  to client  $v_j$ . The set of links/arcs can be called as the set of demands addressed to the supply system.

For all  $i = 1, 2, \dots, m$  production centers and all  $j = m+1, m+2, \dots, m+n$  clients, the conditions regarding quantity distribution are:

- exclusive direct, when all demands are served exclusive direct, without collaborative consolidation within the hub,

$$q_{ij}^d = \begin{cases} q_{ij}, & \text{if } (v_i, v_j) \in Ad; \\ 0, & \text{otherwise;} \end{cases} \quad \text{and}$$

- with the help of consolidation within collaborative hubs

$$q_{ij}^h = \begin{cases} q_{ij}, & \text{if } (v_i, v_j) \in Ah; \\ 0, & \text{otherwise.} \end{cases}$$

The total goods quantity in collaborative system meet the equilibrium conditions ("closing") of flows:

$$\text{- producer/origin, } \tilde{q}_i^h = \sum_{j=m+1}^{m+n} q_{ij}^h, \quad \tilde{q}_i^d = \sum_{j=m+1}^{m+n} q_{ij}^d$$

- client/destination,  $\tilde{q}_j^h = \sum_{i=1}^m q_{ij}^h$ ,  $\tilde{q}_j^d = \sum_{i=1}^m q_{ij}^d$

Modelling hypothesis are as follows:

- the fleet is homogenous and non-restrictive with vehicles of  $Q$  capacity;
- for all  $(v_i, v_j) \in A$  pairs,  $q_{ij} \leq Q$ ;
- direct delivery is allowed only if the volume of goods subject to the transport uses the full capacity of the vehicle (or a loading usage of 80% of its capacity, reasons related to the heterogeneity of goods in terms of the volume occupied), otherwise using hub consolidation.
- the transport cost for a demand within the graph is directly proportional to the distance between the two nodes that bound the link (transport network is uncongested);
- the fixed operating cost within the collaborative consolidation hub are null;
- the variable operating costs are part of the transport costs for entering and exit the hub;
- for the consolidation hub structure there is a variable unit cost  $k$  to handling each arrived loading unit from an origin point, imposing to add a  $k/2$  unit cost for all links incident to the hub.

The problem lies in determining the transport routes (from which some will be direct relations supplier-customer, while others will be delivered via the consolidation hub) in order to minimize the total travel distance of vehicles, meaning that the disjoint sets must be determined:  $Ad$  - subset of demands associated to direct distributions and  $Ah$  - the subset of demands associated with the consolidation hub distributions and, also, the total benefits associated, measured in total length in network, in km.

## 2.2. Solving algorithm

To determine the optimum partition of the  $A$  set  $\{Ad, Ah\}$ , describing the hybrid distribution, the following steps are covered:

(a) suppose that  $Ad = A$  and  $Ah = \emptyset$ , when the problem of determining the optimum routes is solved for the case of exclusive direct supply for all customers' demands;

(b) then, suppose that  $Ad = \emptyset$  and  $Ah = A$ , when the problem of determining the optimum routes is solved for the case of supply with consolidation hub for all customers' demands;

(c) the third step supposes the best of the last solutions as initial solution and with the help of the heuristic algorithm this solution is improved by linking some demand relations from direct distribution to the consolidation hub distribution, or vice versa, as appropriate, considering the initial solution. The solution thus obtained will be better than those two solutions identified for

exclusive supply systems. The heuristics algorithm for the supply efficiency problem can be formalized as follows:

**Step 1.** Solving the problem of exclusive direct supply: for  $i = 1, 2, \dots, m$  a capacitated vehicle routing problem is developed (CVRP) using the Clarke-Wright heuristic algorithm [4] with the starting point located in  $v_i$  to serve  $n$  clients with demands  $q_{i,m+1}^d, q_{i,m+2}^d, \dots, q_{i,m+n}^d$ . The result is measured by the total weighted length/benefit  $Z^d$  (total km, direct supply)

**Step 2.** Solving the problem of exclusively collaborative structure supply (with consolidation hub), with two sequences. Sequence 1 develops a CVRP problem using the same algorithm, with the starting point located in the collaborative consolidation centre  $v_0$  to collect from the  $m$  producers the total goods quantities  $\tilde{q}_1^h, \tilde{q}_2^h, \dots, \tilde{q}_m^h$ . Sequence 2 solves the CVRP problem using the same algorithm with the starting point located in  $v_0$  hub to deliver to the  $n$  clients (processors or their supply platforms) the total demands  $\tilde{q}_{m+1}^h, \tilde{q}_{m+2}^h, \dots, \tilde{q}_{m+n}^h$ . The result is measured by the total weighted length/benefit  $Z^d$  (total km, exclusive consolidation hub supply).

**Step 3.** *Choosing the current solution.*

If  $Z^d \leq Z^h$ , then direct delivery solution will be chosen as initial solution for further improvement, when, at every iteration will become current solution. In this case  $Ad = A, Ah = \emptyset$ . Otherwise, the exclusive collaborative solution of supply with consolidation hub will be chosen as initial solution with  $Ah = A, Ad = \emptyset$ . So, let  $Z = \min\{Z^d, Z^h\}$ , as a measure of the initial solution and note the current solution  $Z^m \leftarrow Z$ , the value of the best solution obtained till the current iteration.

**Step 4.** *Improvement of the current solution*

*Case (1):* If  $Z^d \leq Z^h$ , then for every  $(v_i, v_j) \in Ad$  pair the  $c_{ij}^d$  - „savings” in total length, obtained from the value of the current solution is determined by transferring the demand relation  $(v_i, v_j)$  from the  $Ad$  set to  $Ah$ . Step by step, every  $(v_i, v_j)$  relation will be transferred for a positive  $c_{ij}^d$  from the set of direct distribution to the one of consolidation hub distribution, iteratively. The set of demand relationships for the two categories of exclusive distribution that can be further used on solving the hybrid supply can be written:

$$Ad \leftarrow Ad \setminus \{(v_i, v_j) | c_{ij}^d > 0\} \quad \text{si} \quad Ah \leftarrow Ah \cup \{(v_i, v_j) | c_{ij}^d > 0\},$$

*Case (2):* If  $Z^d > Z^h$ , then for every  $(v_i, v_j) \in Ah$  demand relationship  $c_{ij}^h$  - „savings” in total length, obtained from the value of the current solution is determined by transferring the demand relation  $(v_i, v_j)$  from  $Ah$  to  $Ad$ . So, every  $(v_i, v_j)$  relation will be step by step transferred for a positive  $c_{ij}^h$  from the set of consolidation hub distribution to the one of exclusive direct distribution,

iteratively. The set of demand relationships, according to the gains obtained, can be written:

$$Ah \leftarrow Ah \setminus \{(v_i, v_j) | c_{ij}^h > 0\} \text{ si } Ad \leftarrow Ad \cup \{(v_i, v_j) | c_{ij}^h > 0\}.$$

**Step 5.** Solving the problem of hybrid supply whose demand is divided into the two partitions of the set relations  $A$ ,  $\{Ad, Ah\}$ . It involves solving an actual routing sub-problems related to direct supply for supplier-customer pairs belonging to  $Ad$  and a routing sub-problems related to the exclusive collaborative hub-and-spoke delivery for the pairs supplier-customer belonging to  $Ah$ . Let  $Z'$  the result of this solution.

**Step 6.** If  $Z' < Z$  (the new solution is better than the previous one) then this will be remembered as the current solution:  $Z \leftarrow Z'$ ; otherwise, we will change the direct supply with the hub-and-spoke one, reconsidering  $Z$  the current solution. In case  $Z < Z^m$ , then the new current solution is the best solution so far obtained and retained  $Z^m \leftarrow Z$ . Considering this „best solution" as the current solution, the sequence of the steps from the fourth one is repeated until, after a number of  $N$  consecutive iterations, no significant improvements in results will be obtained, the algorithm stopping.

Steps 1-3 of the heuristic algorithm lead to the initial solution, while steps 4-6 represent a procedure for improving this solution. Values of the hybrid supply problem solutions, the problem of exclusive direct supply and exclusive collaborative supply problem based on the hub-and-spoke model obtained through this algorithm are  $Z^m$ ,  $Z^d$ , and  $Z^h$ .

In case (1) from Step 4,  $c_{ij}^d$  is the net „savings" gained from the value of the current solution by moving the  $(v_i, v_j)$  demand relationship from  $Ad$  to  $Ah$ . Similarly, in case (2) from Step 4,  $c_{ij}^h$  is the net „savings" gained from the value of the current solution by moving the  $(v_i, v_j)$  pair from  $Ah$  to  $Ad$ . The "savings"  $c_{ij}^d$  or  $c_{ij}^h$  are being recalculated at every iteration with the following relation:

$$c_{ij}^d = p_{ij}^d - \varphi_{ij}^h, \quad (1)$$

where  $p_{ij}^d$  is an estimation of the total „economies" gained in the situation where  $q_{ij}$  - the quantity necessary to be delivered from producer/supplier  $v_i$  to client/supply center  $v_j$  is eliminated from the subset of pairs associated to exclusive direct distributions (trip length is shortened);

$\varphi_{ij}^h$  - is an estimation of costs increase when the  $q_{ij}$  demand is added to the subset of pairs associated to collaborative hub-and-spoke distribution (one collecting route to the hub and/or one delivery route from the hub to the client increase).

Every time a modified CVRP routing problem for the hybrid distribution model is required to identify to which route  $q_{ij}$  demand will be added. To avoid solving at every iteration a CVRP problem, suppose  $\varphi_{ij}^h$  equal to the economies

gained by eliminating from transportation the  $q_{ij}$  demand of the exclusive collaborative supply problem. This value is being determined in Step 2.

Similarly:

$$c_{ij}^h = p_{ij}^h - \varphi_{ij}^d, \quad (2)$$

where  $p_{ij}^h$ , and  $\varphi_{ij}^d$  are similar to  $p_{ij}^d$  and  $\varphi_{ij}^h$ . For the same reasons  $\varphi_{ij}^d$  can be approximated with the savings achieved by the elimination from transportation of the  $q_{ij}$  demand in the exclusive direct supply problem. This value is calculated in Step 1.

### 2.3. Assessment of the savings obtained by passing a demand from one type of exclusive supply to another

For the calculation of the savings due to the transfer of some demands from one exclusive supply method to another,  $p_{ij}^d$  and  $p_{ij}^h$ , the usage of the model described by Aykin, T [1] is considered, with the following notations:

$ni$  = number of direct routes associated to the supplier  $v_i$  ( $i = 1, 2, \dots, m$ );

$\tau_i^r$  = length of a direct route  $r$ , from the multitude of exclusive direct supply routes related to the supplier  $v_i$ , ( $i = 1, 2, \dots, m$ ;  $r = 1, 2, \dots, ni$ );

$s_{ij}^r$  = value of the transport distance decrease obtained by removing  $v_j$  client and its demand on the exclusive direct supply route  $r$  related to the supplier  $v_i$ , ( $i = 1, 2, \dots, m$ ;  $j = m+1, m+2, \dots, m+n$ ;  $r = 1, 2, \dots, ni$ );

$n^{col}$  = number of collecting routes from producers/supplier to the hub/collaborative load consolidation centre;

$n^{livr}$  = number of supply routes from the hub/collaborative load consolidation centre to clients/supply platforms;

$q^{col, r}$  = goods quantity collected on the  $r$  route in the collaborative supply system with hub-and-spoke structure ( $r = 1, 2, \dots, n^{col}$ );

$q^{livr, r}$  = goods quantity distributed on the  $r$  route in the collaborative hub-and-spoke supply system ( $r = 1, 2, \dots, n^{livr}$ );

$\tau^{col, r}$  = length of the collecting route  $r$  ( $r = 1, 2, \dots, n^{col}$ );

$\tau^{livr, r}$  = length of the supply route  $r$  ( $r = 1, 2, \dots, n^{livr}$ );

$R_i^{col}$  = set of collecting routes containing quantities shipped by the producer/supplier,  $v_i$ , ( $i = 1, 2, \dots, m$ );

$R_j^{livr}$  = set of supply routes containing quantities for the client/its supply platform,  $v_j$ , ( $j = m+1, m+2, \dots, m+n$ ).

First stage - taking into account the cost reductions achieved by eliminating from the subset of direct relationships the transport route between nodes  $v_i$  and  $v_j$  with their demand  $q_{ij}$ . Denote  $\rho(i)$  the direct supply route from supplier  $v_i$  to customer  $v_j$  of the demand  $q_{ij}$ .



Let the set of direct supply relationships on route  $\rho(i)$  be  $\mathbf{A}_{\rho(i)}^d = \{(vi, vj) \mid \text{demand } q_{ij} \text{ is shipped on } \rho(i) \text{ route}\}$ .

The advantage of dropping the  $q_{ij}$  quantity from  $\rho(i)$  direct supply route can be written:

$$p_{ij}^d = \frac{s_{ij}^{\rho(i)} \cdot \tau_i^{\rho(i)}}{\sum_{k.s.t.(vi,vj) \in \mathbf{A}_{\rho(i)}^d} s_{ij}^{\rho(i)}} \quad (3)$$

Costs reduction when demand  $q_{ij}$  is eliminated from the consolidation hub supply system is:

$$p_{ij}^h = q_{ij} \left( \frac{\sum_{r \in \mathbf{R}_i^{col}} \tau^{col,r}}{\sum_{r \in \mathbf{R}_i^{col}} q^{col,r}} + \frac{\sum_{r \in \mathbf{R}_j^{livr}} \tau^{livr,r}}{\sum_{r \in \mathbf{R}_j^{livr}} q^{livr,r}} \right) \quad (4)$$

Finally, we analyze the cost of introducing the  $q_{ij}$  demand in a existing direct supply route and the gains obtained from removing this demand from the hub-and-spoke supply system. As it was already stated:

$$\varphi_{ij}^d = p_{ij}^d \quad (5)$$

$$\varphi_{ij}^h = p_{ij}^h \quad (6)$$

Similar to the previous notations used for the hybrid supply problem formalization, for the two types of exclusive supply can be written:  $\hat{\mathbf{n}}_i, \hat{\mathbf{\tau}}_i^r, \hat{\mathbf{s}}_{ij}^r, \hat{\mathbf{n}}^{col}, \hat{\mathbf{n}}^{livr}, \hat{q}^{col,r}, \hat{\xi}^{livr,r}, \hat{\tau}^{col,r}, \hat{\tau}^{livr,r}, \hat{\mathbf{R}}_i^{col}, \hat{\mathbf{R}}_j^{livr}$  and  $\hat{\mathbf{A}}_{\rho(i)}^d$ .

From equations (1), (3), (4) and (6) results:

$$c_{ij}^d = \frac{s_{ij}^{\rho(i)} \cdot \tau_i^{\rho(i)}}{\sum_{k.s.t.(vi,vj) \in \mathbf{A}_{\rho(i)}^d} s_{ij}^{\rho(i)}} - q_{ij} \left( \frac{\sum_{r \in \hat{\mathbf{R}}_i^{col}} \hat{\tau}^{col,r}}{\sum_{r \in \hat{\mathbf{R}}_i^{col}} \hat{q}^{col,r}} + \frac{\sum_{r \in \hat{\mathbf{R}}_j^{livr}} \hat{\tau}^{livr,r}}{\sum_{r \in \hat{\mathbf{R}}_j^{livr}} \hat{q}^{livr,r}} \right)$$

From the equations (2), (3), (4) and (5) results:

$$c_{ij}^h = q_{ij} \left( \frac{\sum_{r \in \hat{\mathbf{R}}_i^{col}} \hat{\tau}^{col,r}}{\sum_{r \in \hat{\mathbf{R}}_i^{col}} \hat{q}^{col,r}} + \frac{\sum_{r \in \hat{\mathbf{R}}_j^{livr}} \hat{\tau}^{livr,r}}{\sum_{r \in \hat{\mathbf{R}}_j^{livr}} \hat{q}^{livr,r}} \right) - \frac{\hat{s}_{ij}^{\rho(i)} \cdot \hat{\tau}_i^{\rho(i)}}{\sum_{k.s.t.(vi,vj) \in \hat{\mathbf{A}}_{\rho(i)}^d} \hat{s}_{ij}^{\rho(i)}}$$

Given that this model considered distributions with pre-established schedule, by applying the heuristics algorithm proposed the optimal type of supply for every demand is obtained (direct supply or hub-and-spoke system) and also the vehicles' routing between the nodes' graph.

### 3. Application of the hybrid supply model for Bucharest metropolitan area

The hybrid supply model is achieved for a logistics network retailer within the metropolitan area of Bucharest, whose suppliers,  $m = 20$ , and its supply centers,  $n = 4$ , are located as in Figure 4. The network consolidates the goods through a single collaborative hub. The shortest road distances matrix calculated

on the road network of the metropolitan area of Bucharest, between the  $m + n + 1$  nodes of the graph is shown in Table 1. A sequence of demands between producers and customers is presented in Table 2.

Transportation is realized by a homogeneous trucks fleet with  $Q = 10$  tons capacity each.

Table 1

**Matrix of the shortest distances calculated on the public roads for the logistics network within the metropolitan area of Bucharest (distance in km)**

Distribution centers Suppliers	CD1	CD2	CD3	CD4	Consolidation hub
F1	11.7	10.9	21.4	19.5	13.5
F2	21.7	18.1	14.3	15.6	25.4
F3	21.1	18.2	10.9	12.8	26.4
F4	15.2	11.9	21.6	20.1	17.1
F5	19.1	13.4	20.4	19.6	21.1
F6	25.2	16.1	17.3	18.5	26.2
F7	26.3	22.6	18.8	20.1	30.6
F8	32.1	31.1	23.2	20.2	35.4
F9	18.2	14.8	9.7	5.9	21.9
F10	10.7	15.9	19.4	17.8	11.8
F11	9.4	16.4	26.7	24.5	6.3
F12	14.6	21.7	31.1	29.4	10.9
F13	13.6	20.9	30.9	29.3	10.2
F14	16.7	23.9	33.1	31.1	12.8
F15	19.2	26.4	35.6	33.6	15.3
F16	30.6	37.7	47.1	45.4	26.9
F17	20.1	27.4	37.4	35.8	16.7
F18	14.7	19.9	23.4	21.8	15.8
F19	24.6	31.9	41.9	40.3	21.2
F20	15.1	22.4	32.4	30.8	11.7
CD1	--	--	--	--	5
CD2	--	--	--	--	11.9
CD3	--	--	--	--	22.4
CD4	--	--	--	--	19.8

Vehicle routing was done in Microsoft Excel with CPLEX optimization engine. For the goods demands sequence in Table 3 the following results are obtained:

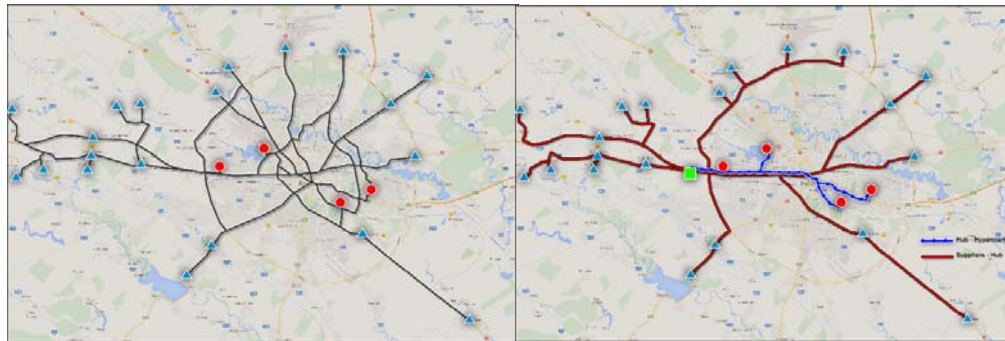
- for the exclusive direct distribution, the total route length is  $Z^d = 1810.2$  km (Figure 4a);
- for the exclusive consolidation hub distribution, the total route length is  $Z^h = 1522.9$  km (Figure 4b);

The initial solution adopted is the exclusive consolidation hub distribution. In the next five iterations, improvement of the total route length is obtained by hybrid supply: total route length is = 1476.7 km (Figure 5).

Table 2

A sequence of demands between producers and customers (tons/day)

Distribution centers Suppliers	CD1	CD2	CD3	CD4
F1	9.7	3.2	0.3	9.3
F2	2.9	0.6	2.8	7.7
F3	6.5	1.9	1.0	0.6
F4	1.3	1.8	9.7	2.4
F5	2.1	1.8	8.4	8.5
F6	7.3	4.6	7.4	7.2
F7	1.5	7.8	3.6	1.4
F8	4.5	4.2	4.0	5.5
F9	0.3	1.2	6.9	3.1
F10	7.4	2.7	6.0	4.9
F11	6.5	9.9	8.2	6.4
F12	4.8	2.7	7.5	9.6
F13	1.4	2.3	6.1	2.2
F14	2.7	6.6	8.4	2.6
F15	4.1	8.2	2.3	9.0
F16	4.7	0.1	7.2	8.4
F17	1.3	7.1	1.8	9.4
F18	8.1	8.3	9.8	2.9
F19	1.5	1.3	6.3	4.1
F20	9.6	8.2	1.8	4.6



a)

b)

Fig. 4. Locations of production centers (blue triangle), supply centers (red circle) and consolidation hub (green square): a) exclusive direct supply; b) exclusive hub-and-spoke supply.

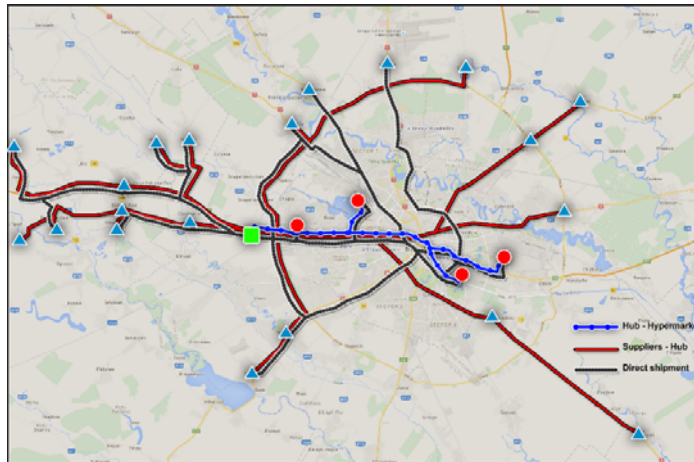


Fig. 5. Representation of the hybrid supply solutions (direct routes and routes with hub operations) for the studied demand sequence

#### 4. Discussion and conclusions

To assess the quality of the heuristic algorithm results random data regarding the demand is generated (quantities between points of origin/producers and destinations/clients) using Visual-Basic software. For all the generated demands the following performance indicators are used:

- relatively average "advantages";
- percent of cases that lead to "advantages".

Consider the general case in which each customer can order goods from any of the suppliers. The daily supply demand (order) of client  $v_j$ ,  $q_{ij}$ , from the  $v_i$  supplier is randomly generated within a range  $[a, b]$  where  $a$  and  $b$  are numerical values given by the limits of the used transport means. The location of the collaborative consolidation centre is considered known (in the application considered, it is Tibbett Logistics Park, Chiajna, Ilfov County).

A total number of 14400 cases of the supply problem for the considered area and the  $20 + 4 + 1$  nodes of the demands graph were generated.

For each of these cases the heuristics algorithm presented is applied to obtain  $\Delta$ ,  $\Delta_{\text{direct}}$  and  $\Delta_{\text{hub}}$ ; then, the relative "advantages" obtained by applying the mixed delivery system are analyzed, compared to the two exclusive delivery systems,  $(\min \{ \Delta_{\text{direct}}, \Delta_{\text{hub}} \}) / \Delta$ .

In all cases considered in this experiment the relatively average "advantages" are in the range of 3.9% to 26.2% compared to the results from the two exclusive delivery systems, when the percentage of cases where "advantages" are obtained is 71.3% (Fig.6).

The two exclusive delivery systems can be considered as extreme cases of the mixed delivery system. Comparing individual, this mixed system recorded a 10.1% relative average "advantage" at a rate of 76.3% compared to exclusive direct supply system and 11.5% relative average "advantage" at a rate of 85.4% compared to exclusive hub-and-spoke delivery system.

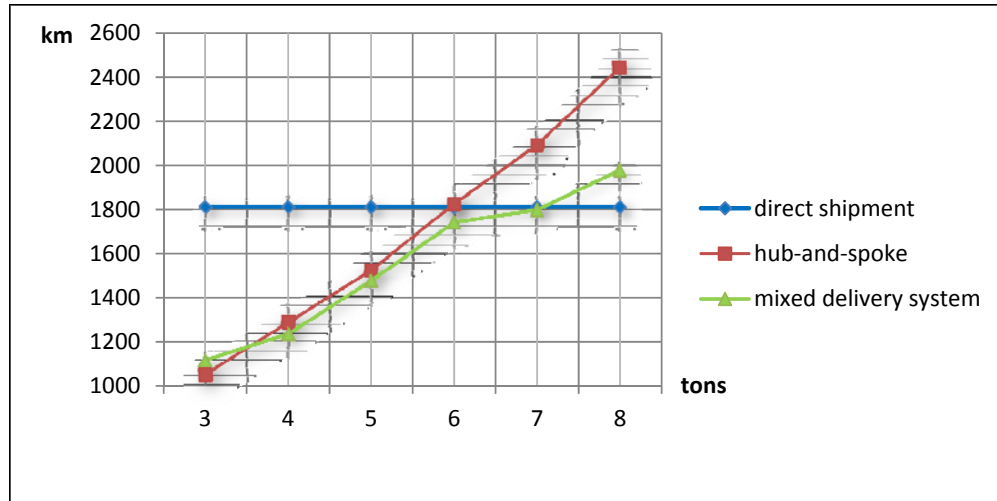


Fig. 6. Total transport distance dependence, in every type of supply, on the average network quantities, in the range of considered demand variation

One can note that for an average supply quantity less than 7 tons of goods on the network layer, exclusive direct distribution records results are superior to the other two types of supply.

For an average supply quantity less than 3 tons of goods on the network layer, exclusive consolidation hub distribution records results superior to the other two types of supply. Overall, these two cases are rarely met in reality as the distribution of large quantities of goods is very heterogeneous, in which case it is obviously advantageous to use the hybrid supply system based on collaborative relationships between the actors of the logistic chains.

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