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Queuing theory models used for port equipment sizing

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Abstract. The significant growth of volumes and distances on road transportation led to the necessity of finding solutions to increase water transportation market share together with the handling and transfer technologies within its terminals. It is widely known that the biggest times are consumed within the transport terminals (loading/unloading/transfer) and so the necessity of constantly developing handling techniques and technologies in concordance with the goods flows size so that the total waiting time of ships within ports is reduced. Port development should be achieved by harmonizing the contradictory interests of port administration and users. Port administrators aim profit increase opposite to users that want savings by increasing consumers' surplus. The difficulty consists in the fact that the transport demand – supply equilibrium must be realised at costs and goods quantities transiting the port in order to satisfy the interests of both parties involved. This paper presents a port equipment sizing model by using queuing theory so that the sum of costs for ships waiting operations and equipment usage would be minimum. Ship operation within the port is assimilated to a mass service waiting system in which parameters are later used to determine the main costs for ships and port equipment.

1. Introduction

Reducing the waiting time of vessels in ports has a special importance from the economic point of view, importance derived from the need to reduce or even eliminate demurrage resulting from failure of the operation times provided in the charter contracts.

An increased waiting time leads to a significant port costs increase, as well as of those cost related to vessels and services provided to ship owners lose quality by the appearance of additional congestion costs [1].

When planning a port development, the commercial interests of port administrations and the public interests of users have to be harmonized. The major commercial interests of ports are the full investment costs recovery and/or obtaining profit. The public interest is linked to the economies experienced by users (users increasing surplus) that might lead to macro-economic growth [2].

Ports are nodes on a route network that connect the origins and destinations of goods' flows. The demand for port services is determined by the competition among the routes. More routes might be used for container transportation from an origin to a destination. Some routes might use maritime transportation for long distances and terrestrial transportation for short distances that might lead to transportation cost reduction but also to longer transport times, while others, the opposite. Models including costs and durations become more complicated when port costs are added to the problem. Some compromises affect the decision of route choosing. It is assumed that a carrier selects the route that minimizes the sum of durations and transport costs between origin and destination, taking into



consideration the operation costs within the port that are paid to the port administration. Each route is supposed to use only one port.

In order to consider the uncertainty regarding the factors that determine carriers' route choosing, a logit model might be used [3]. The aggregation of all the containers processed in a certain port of a transport network determines the transport demand that has to be satisfied by the port.

Harmonizing the public interest (particularly, consumers' surplus) with the commercial interests of ports (particularly, the investment rate of return) is used as fundament for port capacity development. They serve as entries to determine the solution that simultaneously checks two conditions:

1. the optimal sizing of port development;
2. the period for investment return.

The planning problem might be solved by the equilibrium between demand and supply [4]. The demand curve is supposed to change in time. Actually, the continuous change of goods' flows and port congestion might reject the analyze using the demand-supply equilibrium. The equilibrium hypothesis supports solid marks development to analyse the impact on competition and port development.

Both the supply and the demand might be expressed as unit generalized cost (e.g. EUR/TEU). Port development will lead to simultaneously changes of demand and supply for port services. A description of this change, in terms of generalized costs, is used to evaluate the impact of port development [3]. A theoretical shape (a typical one) of this change is presented in Figure 1.

For an equilibrium set at a certain capacity K_0 , the supply curve MC increases with the growth of transit quantity because of the congestion costs within the port. The equilibrium between the $MC(Q, K_0)$ supply and $D(Q)$ demand is set in point Q^* (Fig. 1 a).

If by a competitor port new routes are used, the demand for the port considered falls due to redistribution of the flow of goods on the network. As it is supposed to happen for every port, the demand curve changes from $D(Q)$ to $D'(Q)$. As a consequence, the new equilibrium between demand and supply is set at a new value lower than Q^* (Fig. 1b). If the port has to handle such a situation, a development of port capacities might be realized so that, in a certain manner, the lost demand would be regained.

Considering port capacity growing from K_0 to K_j , the port users congestion cost might decrease, leading to surplus for users. Low congestion costs bring low total generalized costs of routes that also interest the port in case, leading to increased attractiveness among carriers for the routes using the initial port, determining the shifting of the equilibrium point to , value greater than . In practice, this increase might be bigger than the initial loss as the developed capacity might also affect more routes and so, gathering more ships for operating.

The situation after changing the demand from $D(Q)$ to $D'(Q)$ is considered as the reference situation of equilibrium („of doing nothing”) in order to evaluate the development strategy.

If $D'(Q)$ is the demand curve, and are the generalized cost for the new equilibrium with the developed capacity and for the reference situation, then the shaded area is the users surplus growth gained from port development.

In the above approach regarding the demand-supply interaction, the difference between the private marginal cost and the social marginal cost is not considered. This aspect is though necessary when the congestion price is included in the planning problem.

Solving the rational sizing of port equipment is difficult both because of the endogenous factors and of the exogenous ones that affect the whole port system. These often lead to variations of traffic or equipment productivities and to variable usage solicitations. The causes of variable usage solicitations might be grouped into three categories: economic, techniques and organizational [5].

Reducing the consequences of variable exploitation solicitations is achieved by maintaining supplementary capacities used during high peaks. The disadvantage of this solution is that the installation is not fully used most of the time and investments are difficult to be returned.

Trying to find the best solutions for the optimal structure of installations with variable solicitations several steps have been taken. First, the planners chose the size of installations based on experience, comparing with similar existing installations that had maximum return. Later, mathematical models

that took into account an equivalent solicitation of equipment were developed. These models do not obey the economy principle, as calculation takes into account an average solicitation that does not correspond to a real situation. Still, the appearance of these models might be seen as a progress in eliminating supplementary constructions and installations [6].

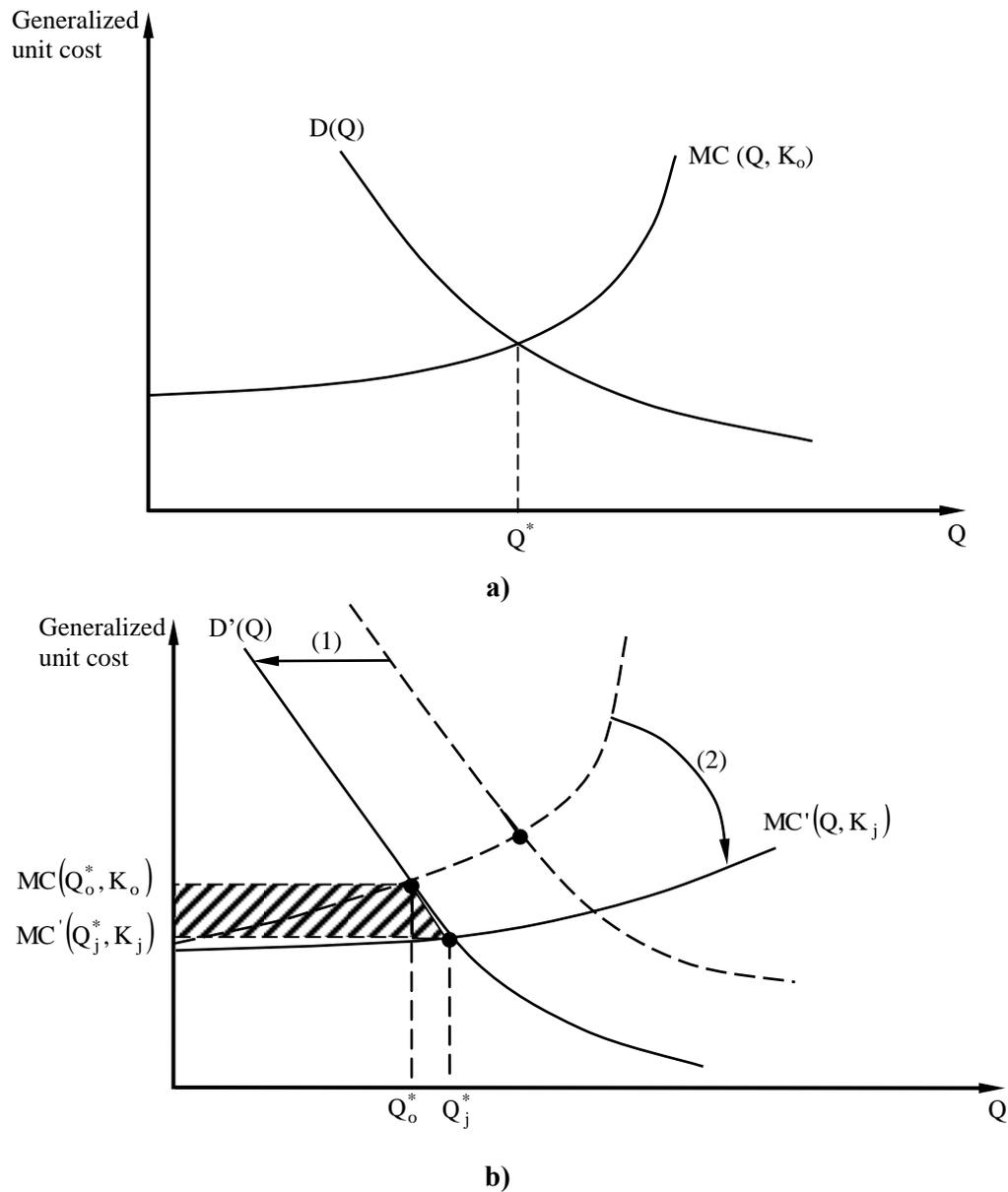


Figure 1. Port services demand-supply interaction [3]: (1) demand modifies (2) capacity development.

Optimum results are obtained by computer simulation of the usage process of port installations [7] and by using Bayesian networks for port activity planning [8]. The difficulties of simulation are represented by the big effort demanded for a small precision gain and by the big amount of initial data necessary.

Specialists agreed that the only viable criterion for an optimal port sizing is the economic one that aims maximizing the social benefit [9], in this case, the minimum sum of costs for ships and berths.

2. Analytic sizing

The system consisting of port roadstead where the ships are waiting, berths for loading and unloading ships arriving in port for these operations, is a mass service waiting system [10].

The problem to be solved is linked to the modifications that have to be brought to the system (building new berths, reducing the service time by modernization of the berth installations etc.) in order to reduce penalties. Also, a rational sizing in relation to the goods quantities operated, constructions and installations with variable usage solicitations must be realised [11].

Waiting phenomena supposes optimizing some functions or system parameters. Most often, the economic function of the sum of waiting costs for demands arrived for service and also for the service stations is optimised [12]. Figure 2 presents the graphical variation of the total specific costs for port system (the cost for ships waiting service and the inactivity cost of berth) that has a minimum in point B. This minimum is specific to a traffic volume less than the minimum of the port specific costs (point A), because, in most of the cases, ships waiting costs are greater than the downtime costs of port facilities (the usual ratio is 4/1) [5]. The minimum value of port operational cost might be obtained by linear programming [13]. Most often, along with the economic function, restrictions are also imposed, obtaining the optimal solution being achieved by calculating the objective function $F(s)$ for different values of the number of service stations (s).

The economic function is expressed mainly as a statistic average of costs brought by demands waiting service and installations downtimes in a T time interval:

$$F(s) = (c_1 \bar{n}_a + c_2 \bar{\Phi})T = [c_1 \lambda \bar{t}_a + c_2 (s - \rho)]T \quad (1)$$

where c_1 is the time unit cost for waiting demand;

- average number of units within the queue;

c_2 – time unit cost for service stations downtimes;

- average number of unoccupied stations;

λ - average arrivals intensity;

- service intensity factor (number of demands arriving in the system while an unit is in service);

μ - average service intensity;

- average queue waiting time..

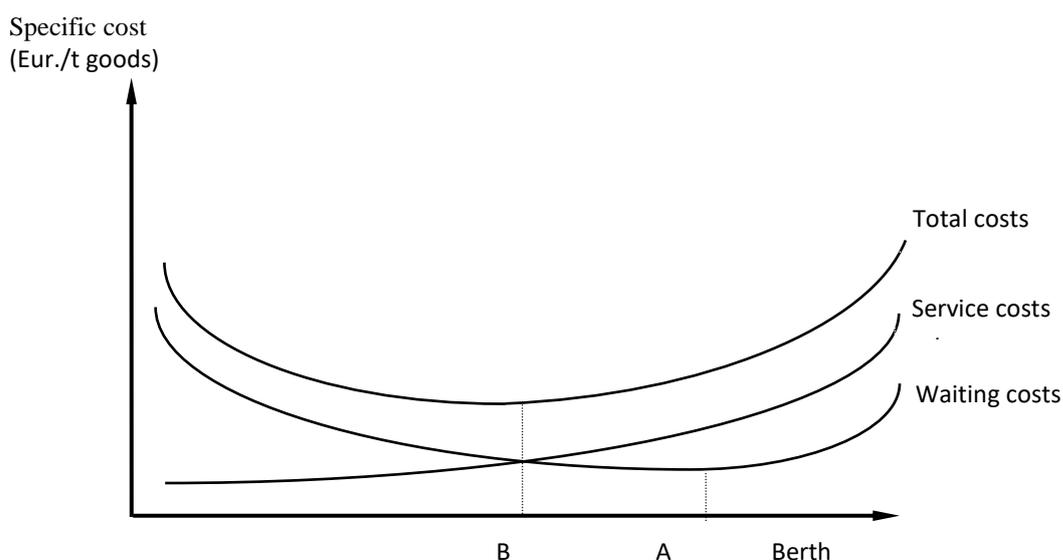


Figure 2. Total specific costs of the ships and berths system.

F(s) is determined for more values of (s) and the minimum value is retained. When other restrictions are imposed they would be taken into consideration (for example, the number of units in the queue not to exceed a certain limit or the probability of a demand waiting more than a specific number of minutes). The system of berths and ships arrived for operations is considered a mass waiting service system whose parameters are to be determined according to the arrival and service discipline [14], [15].

Within F(s), ρ and \bar{t}_a must be determined. ρ is quite simple to be determined, while \bar{t}_a requires thorough calculation and a comprehensive study of the waiting phenomenon

3. Case study

For the model development the arrivals of ships in the port were first studied (for a period of 20 days). The situation is presented in Table 1.

Table 1. Ships arriving in port.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Number of ships arrived	3	4	5	4	1	3	2	6	0	2	3	4	6	6	3	1	4	7	3	2

The number of ships arrived per day varies from 0 to 7. The concordance between the empiric repartition and the theoretical one considered is verified ((M/M/s(∞/FIFO) – Poisson distribution for arrivals and Exponential distribution for service time, with s number of parallel servers, ∞ places in the system and FIFO discipline queuing system) and presented in Table 2.

Table 2. Average and dispersion for the empirical random variable.

n	f(n)	n-m _n	(n-m _n) ² f(n)	P _n = $\frac{(m_n)^n}{n!} e^{-(m_n)}$	$\frac{[f(n) - N \cdot P_n]^2}{N \cdot P_n}$
0	1	-3.45	11.902	0.0317	0.2113
1	2	-2.45	12.005	0.1095	0.0165
2	3	-1.45	6.307	0.1889	0.1602
3	5	-0.45	1.012	0.2173	0.0984
4	4	0.55	1.210	0.1874	0.0169
5	1	1.55	2.402	0.1293	0.9727
6	3	2.55	19.507	0.0743	1.5425
7	1	3.55	12.602	0.0366	0.0981
$N = \sum_n f(n) = 20$			$\sigma^2 = 3.3475$	$\chi^2 = \sum_n \frac{[f(n) - N \cdot P_n]^2}{N \cdot P_n} = 3.1166$	
$m_n = 3.45$					

$$m_n = \frac{1}{N} \sum_n n f(n) = 3.45 \tag{2}$$

$$\sigma^2 = \frac{1}{N} \sum_n (n - m_n)^2 \cdot f(n), \tag{3}$$

where m_n is the average of the empirical random variable;
 σ^2 - dispersion of the empirical random variable.

In order to check the concordance between the empirical and the theoretical random variable χ^2 test is used. If $\chi^2_{\text{calculat}} < \chi^2_{\text{tabelat}}$, then the conclusion that a good concordance exists between the two random variables can be drawn (Table 3).

Table 3. χ^2 Repartition.

Degrees of Freedom	Probability	
	0.05	Article I. .01
1	3.84	6.63
2	5.99	9.21
3	7.81	11.24
4	9.49	13.28
5	11.07	15.08
6	12.59	16.81
7	14.07	18.47
8	15.51	20.09
9	16.92	21.66
.		
.		
20	31.41	37.57

The degrees of freedom might be determined with the relation: $v = n - m - 1$ ($n = 8$ (varies from 0 to 7), m is the number of parameters of the considered theoretical repartition (Poisson repartition has 1 parameter, λ).

$v = 8 - 1 - 1 = 6$ degrees of freedom and, for a safety step. As $\chi^2_{\text{calculat}} = 3.504 < 12.59$ χ^2 is in the confidence interval corresponding to the $p = 0.05 \Rightarrow \chi^2_{\text{tabelat}} = 12.59$ safety index of 0.95 the observed frequency regarding ships arriving in port corresponds to a Poisson law. This fact is also sustained by the ratio between the average and the dispersion of the empirical random variable that is close to the unit.

Similarly, based on observation, the ships service on berths was studied, determining that the service time also follows a Poisson repartition, checked with χ^2 test. The average service time for a ship, determined by statistical study, is 20 hours, and so, the average number of service in the time unit (24 hours a day) is: $\mu = \frac{24}{20} = 1.2$ and $\lambda = \frac{1}{20} \sum_n n \cdot f(n) = 3.45$.

So, $\rho = \frac{\lambda}{\mu} = 2.875 > 1$. For one berth the queue increases to infinite and the system does not work

stationary. For a stationary regime within the system, $\rho < 1$. This happens for a number of berths (s) equal to $3 \Rightarrow \rho^* = \rho / 3 = 0.958 < 1$.

Verifying that the number of berths is appropriate will be achieved by calculating the main elements of the model for $s = 3, 4, 5, 6$ and 7 .

$$\bar{t}_a = \frac{\rho^s}{s \cdot s! \mu (1 - \rho/s)^2} \cdot P(0), \quad (4)$$

$P(0)$ represents the probability that all stations are free or the probability of not having waiting times and is determined with relation (5).

$$P(0) = \frac{1}{\frac{\rho^s}{s!(1-\rho/s)} + \sum_{k=0}^{s-1} \frac{\rho^k}{k!}} \tag{5}$$

Inactivity costs for the system are presented in Table 4.

Table 4. Inactivity costs in case of introduction of a supplementary berth.

s	P(0)	\bar{t}_a [days]	System downtimes D_i [h/day]	Waiting times for arrived ships D_a [h/day]	Costs for hours lost by ships and service stations $C = c_1 \cdot D_a + c_2 D_i$		
					$c_1=500um$ $c_2=300um$	$c_1=500um$ $c_2=500um$	$c_1=300um$ $c_2=500um$
3	0.0097	6.050	3	500.94	251370	251970	151782
4	0.0450	0.337	27	27.90	22050	27450	21870
5	0.0530	0.080	51	6.62	18610	28810	27486
6	0.0550	0.022	75	1.82	23410	38410	38046
7	0.0560	0.006	99	0.49	29945	49745	49647

The inactivity time for the service stations is determined with relation (6):

$$D_i = s \cdot 24 - \lambda \bar{t}_{sv} \tag{6}$$

and the average operation waiting time for ships with relation (7):

$$D_a = \lambda \cdot \bar{t}_a \tag{7}$$

The average service time of the system in one day is obtained by multiplying the number of ships arriving one day with the necessary service time for a ship. Cost c_1 is also known (one hour stationing cost per ship) and c_2 (one hour of inactivity for the system). The probability of waiting for a ship can also be calculated $P(> 0)$

$$P(> 0) = \frac{\rho^s}{s!(1-\rho/s)} P(0) \tag{8}$$

in hypothesis 3, 4, 5, 6, 7, 8 and 9 service stations, as well as the number of ships waiting operations

$$\bar{n}_d = \frac{\rho^{s+1}}{s \cdot s!(1-\rho/s)^2} P(0) \tag{9}$$

The results of calculations were synthetized in Table 5.

Table 5. Waiting probability and average number of ships waiting for operations.

Number of berths	3	4	5	6	7	8	9
$P(> 0)$	0.922	0.455	0.204	0.083	0.031	0.01	0.003
\bar{n}_d	21.21	1.16	0.276	0.039	0.021	0.0056	0.0016

The same problem can be studied by maintaining the number of berths but with ships handling equipment productivity increase so that the service time would reduce from 20 hours to 18, 16, 14, 12, 10 or even 8 hours for each served ship.

The cost of lost hours for ships and berths in case of 3 berths and varying service time is presented in Table 6.

Table 6. Inactivity costs in case of 3 berths and a varying service time.

s	\bar{t}_{sv}	P(0)	\bar{t}_a [days]	System downtimes D_i [h/day]	Waiting times for arrived ships D_a [h/day]	Costs for hours lost by ships and service stations $C = c_1 \cdot D_a + c_2 D_i$		
						$c_1=500um$ $c_2=300um$	$c_1=500um$ $c_2=500um$	$c_1=300um$ $c_2=500um$
3	20	0.0097	6.050	3.0	500.94	251370	251970	151782
3	18	0.0370	1.310	9.9	108.46	57200	59180	37488
3	16	0.0680	0.563	16.8	46.62	28350	31710	22386
3	14	0.1100	0.265	23.7	21.94	18080	22820	18432
3	12	0.1610	0.127	30.6	10.52	14440	20560	18456
3	10	0.2260	0.057	37.5	4.72	13610	21110	20166
3	8	0.3110	0.023	44.4	1.91	14275	23155	22773

The minimum value of losses within the system, for an exact number of 3 berths, for varying cost c_1 and c_2 , is obtained for a service time of ships from 10 to 14 hours.

The cost of lost hours for ships and service stations in case of 4 berths is presented in Table 7.

Table 7. Inactivity costs in case of 4 berths and a varying service time.

s	\bar{t}_{sv}	P(0)	\bar{t}_a [days]	System downtimes D_i [h/day]	Waiting times for arrived ships D_a [h/day]	Costs for hours lost by ships and service stations $C = c_1 \cdot D_a + c_2 D_i$		
						$c_1=500um$ $c_2=300um$	$c_1=500um$ $c_2=500um$	$c_1=300um$ $c_2=500um$
4	20	0.045	0.337	27.0	27.90	22050	27450	21870
4	18	0.067	0.1	33.9	8.28	14310	21090	19434
4	16	0.093	0.073	40.8	6.04	15260	23420	22212
4	14	0.129	0.043	47.7	3.56	16090	25630	24918
4	12	0.175	0.023	54.6	1.91	17335	28255	27873

Table 7 shows that inactivity costs' sensitivity is very little in relation to the unitary cost of waiting for ships and berths. This happens when the number of berths is increased with one unit.

The cost of lost hours for ships and service stations in case of 5 berths is presented in Table 8.

Table 8. Inactivity costs in case of 5 berths and a varying service time.

s	\bar{t}_{sv}	P(0)	\bar{t}_a [days]	System downtimes D_i [h/day]	Waiting times for arrived ships D_a [h/day]	Costs for hours lost by ships and service stations $C = c_1 \cdot D_a + c_2 D_i$		
						$c_1=500um$ $c_2=300um$	$c_1=500um$ $c_2=500um$	$c_1=300um$ $c_2=500um$
5	26	0.0186	0.4000	30.3	33.12	25650	31710	25086
5	24	0.0275	0.2330	37.2	19.30	20810	28250	24390
5	22	0.0384	0.1403	44.1	11.62	19040	27860	25536
5	20	0.0530	0.0800	51.0	6.62	18610	28810	27486
5	18	0.0740	0.0443	57.9	3.67	19205	30785	30051
5	16	0.0986	0.0242	64.8	2.01	20445	33405	33003
5	14	0.1329	0.0118	71.7	0.98	22000	36340	36144
5	12	0.1775	0.0053	78.6	0.44	23800	39520	39432

The minimum value of losses within the system, for an exact number of 3 berths, for varying cost c_1 and c_2 , is obtained for a service time of ships from 24 to 20 hours..

4. Conclusions

Establishing a rational structure for equipment size asked for more calculations of the economic function of the system for different ship service times and number of berths. The calculation was realised varying the number of berths (from 3 to 7), ship service times (from 8 to 26 hours) and the inactivity costs of berths and waiting time costs of ships. Each time the minimum value of the economic function was kept. An exact role for this function was played by the downtime costs of berths and by the waiting costs of the ships. In this case the analyse can be detailed by considering different downtime costs for berths with service personnel or without, knowing that these are affected by salary costs with high weight in the total cost of port equipment usage.

For calculations, the relations provided by queuing theory were used, not always offering a numeric solution that depends on the statistic repartition of arrivals and ships service times.

In the developed case study, increasing the number of berths led to an optimal solution for a higher ship service time. The solutions' sensitivity analyse in relation to the value of the cost for one hour of a ship stationing and system inactivity proved that for 4 berths the importance of the size of these two costs is little in determining the minimum value of the objective function.

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